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APPLICATIONS OF SOME NUMERICAL MODELS
TO PACIFIC NORTHWEST ESTUARIES

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ABSTRACT

Physical-chemical features of some Oregon and Washington estuaries are discussed in relation to existing or potential pollution problems. Available mathematical models of flow and dispersion phenomena are reviewed and applications of a steady-state, a time-variable, and a box model are presented. The effects of estuaries on oceans are illustrated via examples on the Columbia River and Cook Inlet.

INTRODUCTION

→ This paper considers estuarine processes from the deterministic viewpoint and emphasizes numerical solutions of differential equations via the digital computer rather than the analog-hybrid or hydraulic model approach.

It is recognized that the hybrid computer may prove highly useful for large scale, multi-dimensional, diffusion problems especially when these include n^{th} order kinetic reactions. The hydraulic model will remain an extremely useful tool in a variety of situations such as sediment loading, scour, etc. (See Fischer and Holley, 1971, for a discussion of their use in dispersion studies.)

→ Briefly reviewed are some recent modeling efforts on two aspects of the simulation of estuarine processes. These are: (1) the representation of currents, and water levels, and (2) the representation of diffusion and the local change of concentration due to currents, diffusion, and source-sink terms. The source-sink terms represent → p31

waste loadings, bottom demands, growth-death rates, phytoplankton grazing, etc., and are considered more in the light of ecological modeling. Sets of coupled equations are necessary for dealing with ecological processes and these must also include the two aspects listed above; hence, while the modeling of the ecology of a system is usually highly dependent on the physical processes the reverse situation is only rarely true.

Users who develop and apply models must be aware of the numerical pitfalls involved in applying finite difference approximations which are usually involved in the study of real-life pollution problems. They must also be able to represent the fluid motion in complex estuarine waters with its tidal currents and changing water levels; i.e., the advection and diffusion processes which exist. The simulation of these processes requires employment of numerical techniques and consideration of stability and mass and momentum conservation. This paper is directed mainly to the user although numerical and fluid dynamical problems are also discussed.

A summary of the state-of-the-art for simulation of pollution problems and controls in estuaries has been presented by Baumgartner and Callaway (1970). Their paper was based on an assessment of pollution control capabilities performed by the TRACOR Corporation (Ward and Esrey, Ed., 1971) under contract to the Environmental Protection Agency. Emphasis in the above papers was on deterministic models rather than on empirical or stochastic approaches and should be examined for a general view of present day estuarine modeling capabilities.

Estuarine-Riverine Zones

The commonly used definition of an estuary is that it is a semi-enclosed body of water having free connection with the open ocean and whose waters are measurably diluted by fresh water drainage. Some of our Pacific Northwest estuaries, as will be shown, are completely flushed out by fresh waters during periods of high runoff hence, under

a strict application of the above definition, should probably be treated as tidal rivers when such conditions exist.

A large river system, such as the Columbia, has essentially three separate zones: (1) a saltwater portion which, for the Columbia, penetrates a maximum of 20-25 miles upstream of the entrance, (2) a freshwater, current reversal zone upstream of the salt section and, (3) a run-of-the-river section upstream of the latter zone where current direction is always directed seaward but which might exhibit vertical, tidally induced, oscillations, usually on the order of inches. The transition region between successive zones migrates longitudinally and depends on source salinity, tide range and duration, tributary and mainstream flow, and winds. For the Columbia, all three zones have been modeled (Callaway et al., 1969). While the simulation of such a large system results in a rather complex grid for the flow network it does simplify upstream boundary conditions as they may be held constant or allowed to vary with observed tailwater elevations at upstream points (e.g., Bonneville Dam for the Columbia).

The saltwater portion of a coastal river is significant from the point of view of its ecological setting. The dynamics of this zone are complicated by the fact that gravitational convection (see Hansen and Rattray, 1965), due to the density difference between fresh and salt waters, can set up circulation patterns that are difficult to simulate. The so-called 1-dimensional model imposes no horizontal or vertical deviations in velocity or salinity or pollutant. Slight vertical changes in salinity (density) are enough to generate convection currents which will in turn modify the distribution of a pollutant in the estuary. It may be that this contribution to the pollutants' distribution is minor but the modeler must be aware of the existence of such things if only to aid in understanding reasons for discrepancies, if they occur, between model and prototype.

Flow Regimes

If an effluent is discharged to an estuary through a canal or pipe it may approximate a point source. The discharge may be continuous or periodic; the volume of flow and the mass of pollutant discharged may vary at the same or different rates. The velocity of flow and dimension of the orifice may indicate that the momentum of the discharge will be enough to carry it some distance into the receiving water. The density difference between the effluent and the ambient stream and the depth of discharge may also contribute to the path the effluent follows before the ambient currents and turbulence dominate the flow.

In general, three flow regimes exist (see Baumgartner and Trent, 1970): (1) a zone of flow establishment; (2) a zone of established flow, and (3) a zone of drift flow. The first two zones are related to the initial momentum and/or buoyancy of the discharged fluid relative to the ambient fluid. If these zones are small in area it may be possible to approximate them by denoting one or more of the grid areas as sources--(assuming ambient conditions have taken over). If the zones are dominant or the numerical grid unsuitable, then models incorporating these features must be used. Typical of the models which do this are those of Brooks (1959), Okubo (1967), and Koh and Fan (1970).

ADVECTION-DIFFUSION PROCESSES

Specialists in a given field often work around a surprisingly few basic differential equations. In estuarine work we usually concern ourselves with a fixed grid system and predict the local time change of concentration of a substance. The concentration is presumed modified in transit by advection and diffusion; the total amount in a closed system remains constant unless it degrades naturally or is modified by chemical or biological processes.

The so-called advection-diffusion (AD) equation can be written as:

$$(1) \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} - \frac{\partial}{\partial x} (K_x \frac{\partial s}{\partial x}) - \frac{\partial}{\partial y} (K_y \frac{\partial s}{\partial y}) - \frac{\partial}{\partial z} (K_z \frac{\partial s}{\partial z}) = \Sigma S$$

where s = concentration of substance

u = velocity in the x -direction

v = velocity in the y -direction

w = velocity in the z -direction

K_x, K_y, K_z = eddy coefficients

ΣS = sum of the sources and sinks

The terms containing velocity components make up the advective transport part of the equation; terms with eddy coefficients make up the turbulent transport. The source and sink terms define pollutant additions, phytoplankton growth and death rates, etc. If more than one substance enters into a reaction then two or more equations will be coupled through feedback or feedforward mechanisms in the source and sink terms.

Difficulties exist concerning all the terms in the equation; the physical processes are sometimes difficult to express as functions and must be solved numerically (and usually separately) before inclusion

in the equation. This can cause problems relating to the conservation of mass in the numerical process and constitutes a large area of research pursued by numerical analysts and fluid dynamicists.

Presently more intractable, however, is the role of the source and sink terms, both from the viewpoint of the experimentation needed to delimit certain coefficients and the non-trivial mathematical representation of several interacting components in an ecological chain. The TRACOR (1971) report should be referred to here for a more thorough discussion of the above topics.

Recent Work on the AD Equation

Table 1 summarizes the terms included in some, but not all, of the numerical models of the AD process in estuaries. Such a representation can be misleading since the programs for running the models are usually easily expanded or modified to include extra phenomena. For purposes of this exposition the documentation of the programs or descriptive papers available to the author was examined in order to make up the table. The progress indicated in the table in estuary (and/or coastal) models is noteworthy. Starting from Stommel's steady-state, first-order kinetics model, in the relatively computer-free times of 1953 to Thomann's first computer model in 1963 the expansion progressed in the main, to a non-steady state, one-day average, simulation of the BOD-DO system of a very large, complex tidal river. Two-dimensional vertically integrated cases are now handled by Leendertse's model although by no means routinely. Although there is still much to be learned in the vertically integrated case, progress is even more wanting in the representation of the vertical distribution of currents and pollutants.

TABLE 1. SOME RECENT APPROACHES TO MODELLING
OF THE ADVECTIVE-DIFFUSION EQUATION

Author	Local	Advective Transport	Turbulent Transport	Source-Sink and Remarks
		$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} - \frac{\partial}{\partial x} \left(K_x \frac{\partial s}{\partial x} \right) - \frac{\partial}{\partial y} \left(K_y \frac{\partial s}{\partial y} \right) - \frac{\partial}{\partial z} \left(K_z \frac{\partial s}{\partial z} \right) = \Sigma S$		
Stommel (1953)		✓	✓	First order kinetics. Vel. = Q/A. Diffusion from salt conc.; relaxation solution.
O'Connor (1960)	✓	✓	✓	Analytical solution. BOD-D0. Vel. = Q/A. Diffusion as above.
Thomann (1963) (Jeglic, 1967)	✓	✓	✓	BOD-D0. First order kinetics. Vel. = Q/A. Diffusion via eddy exchange coeff.
Shubinski et al. (1965)	✓	✓	✓	BOD-D0. First order kinetics. Vel. from cont. & momentum. Diffusion from scale (depth) of channels.
Okubo (1967)	✓	✓	✓	Analytical solution. First order decay. Includes velocity shear.
Callaway et al. (1969)	✓	✓	✓	As in Shubinski; + temp. Surface heat exchange.
Leendertse (1970)	✓	✓	✓	Vel. from mom. & continuity. Diffusion as a function of current speed, friction, depth + wind-wave relationships. Nth order kinetics.
DiToro et al. (1970)	✓	✓		Box model of phytoplankton dynamics. Michaelis-Menten kinetics.
Shankar and Masch (1970)	✓	✓	✓	Vel. terms based on Reid-Bodine model. Conserv. distribution.
Fisher (1970)	✓	✓	✓	2-D hydraulics by Leendertse model; 1-D by characteristics. First order kinetics. Diffusion by attempt to incorporate shear in 2-D program.

One point should be made in relation to the way equation 1 and the Table express the advective transport. Any correct method of solving a given equation should lead to the same answer. Unfortunately, some seemingly correct finite difference approximations of the continuous processes assumed in the equations can badly represent the intent of the approximation. Illustrative examples are given by Crowley (1968) who has shown the improvement in the use of fourth-order over second-order approximations (referring to the Taylor series truncation) and more significantly, improvement in using the conservation form of the advection equation. The latter consists in writing the equation:

$$(2) \quad \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} = 0$$

as

$$(3) \quad \frac{\partial s}{\partial t} + \frac{\partial(su)}{\partial x} + \frac{\partial(sv)}{\partial y} - s \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0$$

The gain in accuracy is due to the fact that the former equation does not necessarily conserve mass during the iteration process; neither does the latter (exactly) but it is much better behaved. This topic has been discussed rather extensively by Leendertse (TRACOR, 1971); he has also treated the numerical problems involved in both the hydraulic and AD equations.

HYDRAULIC EQUATIONS

The velocity terms in equation 1 are usually computed independently at short time intervals (seconds to minutes), output to a disc or tape and then averaged over a longer time period. The average values (minutes) are input to the equation upon demand. Diffusion terms are expressed as functions of a characteristic scale, as functions of the variance of an observed spreading of material, as a random process, etc. As the advection process is more closely approximated at shorter time intervals the magnitude of the diffusion term becomes less--relative to the advective transport process--and can sometimes be neglected.

Table 2 shows terms in the x-direction equation of motion. As in the table before, some recent reports concerning computation of the velocity terms have been examined as to the assumptions implied by the terms included or excluded. This is not represented as a complete summary; it is meant to relate to investigations whose primary objective was for use in estuaries or which have since been modified for such use.

Trouble arises in the explicit methods of solution (marching methods) because of the short time periods required to ensure stability and the general problem related too in the AD equation concerning mass conservation. In this case momentum conservation can be violated resulting in abnormal velocity and water transport volumes. These problems are quite real but can be modified by judicious selection of computing schemes, such as the alternating direction implicit scheme of Leendertse (1970).

All of the methods above revolve around the primitive equation of motion except for Okubo's. (The continuity equation has to be solved

TABLE 2

EQUATION OF (X-DIRECTION) MOTION EMPLOYED BY
VARIOUS RECENT INVESTIGATIONS

Author	Inertial $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$	Convective $u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$	Potential $+ g \frac{\partial h}{\partial x}$	-	Coriolis fv	+	Bottom Friction $\frac{gu\sqrt{u^2+v^2}}{C^2(h+d)}$	=	Wind $F_x + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\partial p_0}{\partial x}$	Viscous	Atm. Pressure
Hansen, W. ^{1/} (1962)	✓			✓			✓		✓	✓	2/
Shubinski ^{3/} and Scheffey ^{4/} (1966)	✓	✓					$K u u^{4/}$		5/		
Leendertse (1967)	✓	✓	✓		✓		✓		5/		
Reid and Bodine (1968)	✓		✓				✓		✓		
Masch, et al. (1969)	✓		✓		✓		✓		✓		
TRACOR (In Press)	✓		✓		✓		✓		✓		

^{1/} Multi-level models in preparation^{2/} Has been included in large-scale grid models^{3/} Quasi-2-dimensional^{4/} Manning representation rather than de Chezy^{5/} Has been incorporated in other versions

in conjunction with the finite difference approximations. Okubo's is an analytical solution.) Okubo has included lateral and vertical shear in the x-direction velocity term, prompted by the fact that dye releases are usually observed to elongate in the direction of the current rather than spread radially about the center of mass. Inclusion of the velocity shear terms enables one to more nearly simulate instantaneous and continuous source releases (Okubo and Karweit, 1969). As have many things, the original work on the importance of shear (Saffman, 1962) was generated by meteorologists. They have an unfair advantage, of course, in that their observations are sinfully easy to obtain and reproduce compared to oceanographic measurements. This has resulted in their making relatively great advances in entering the 3-dimensional space domain by employing multi-level, global, models of circulation.

FEATURES OF SOME OREGON-WASHINGTON ESTUARIES

Burt and McAlister (1959) reviewed the hydrography of Oregon estuaries and classified them on the basis of their vertical salinity profiles. Hansen and Rattray (1966) have classified estuaries on the basis of circulation and stratification parameters at given cross-sections of an estuary. Dimensionless ratios of net surface current to mean freshwater velocity (u_s/u_f) and top-to-bottom salinity difference ($\delta S/S_0$) are used to exhibit the physical significance of different systems.

Figure 1 shows the classification in terms of the above ratios. The examples given by Hansen and Rattray are replotted with an abbreviated description of the different types (1-4). West coast waters shown are the Columbia (C); Straits of Juan de Fuca (JF); and Silver Bay, Alaska (S). A point for the Yaquina River (Y) at mile 14 has been added to the graph from data collected by the author. The Yaquina data were taken during a 25-hour survey; the stratification parameter equals 0.12, and the circulation parameter is 1.64. The coordinates of the point place the estuary at this river mile, at this time, in type 1b, a case of "...appreciable stratification." The nearly vertical line of dots through the point was obtained by computing individual hourly circulation and stratification parameters in order to obtain an idea of the range of values that might be expected under the prevailing conditions. As can be seen, the dots extend into higher stratification values (as shown by an increase in $\delta S/S_0$) and also range into type 1a "...the archetypical well-mixed estuary in which the salinity stratification is slight...."

Vertical profiles of salinity and current (used in computing the above parameters) and dissolved oxygen are given in Figure 2a which shows

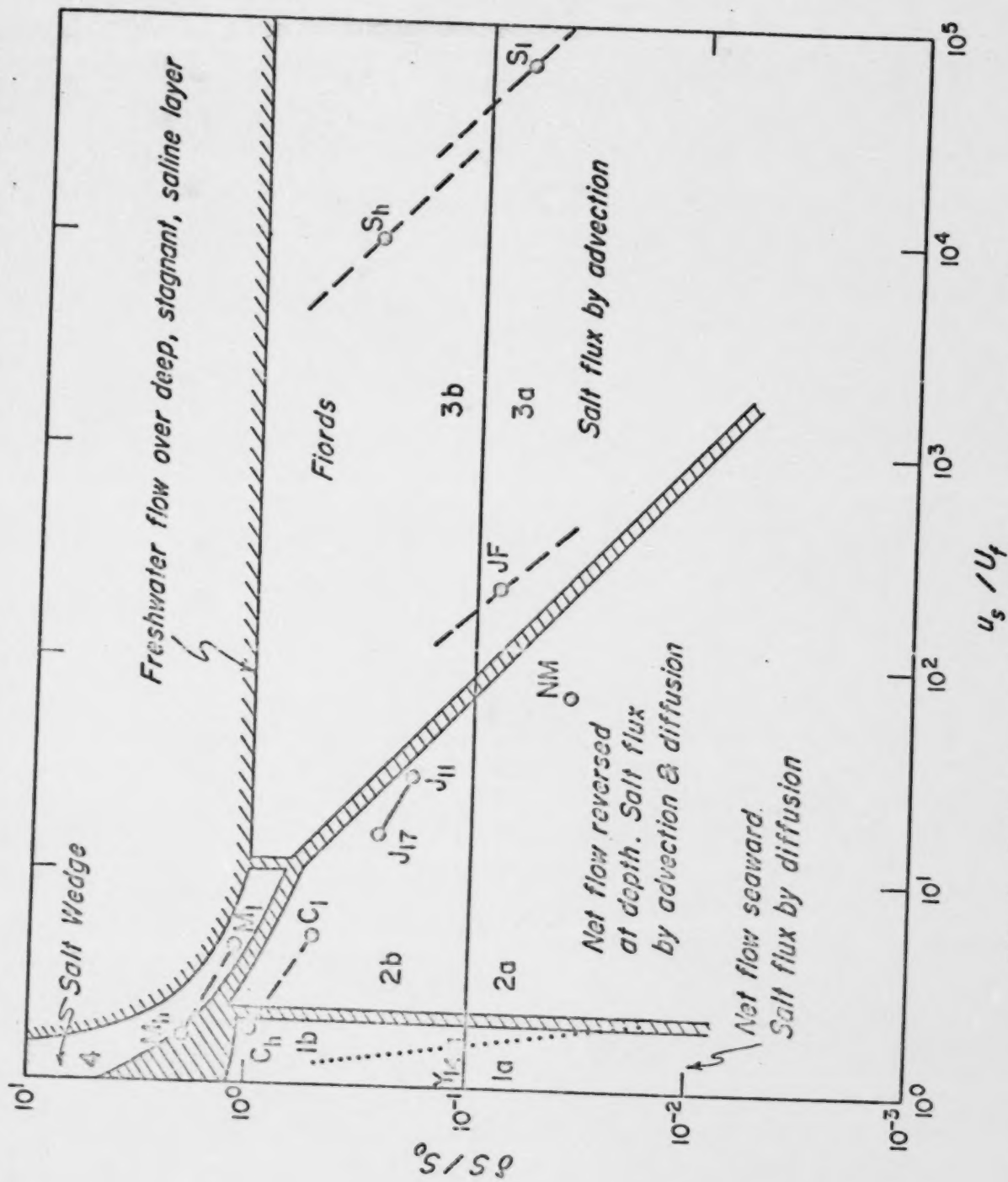
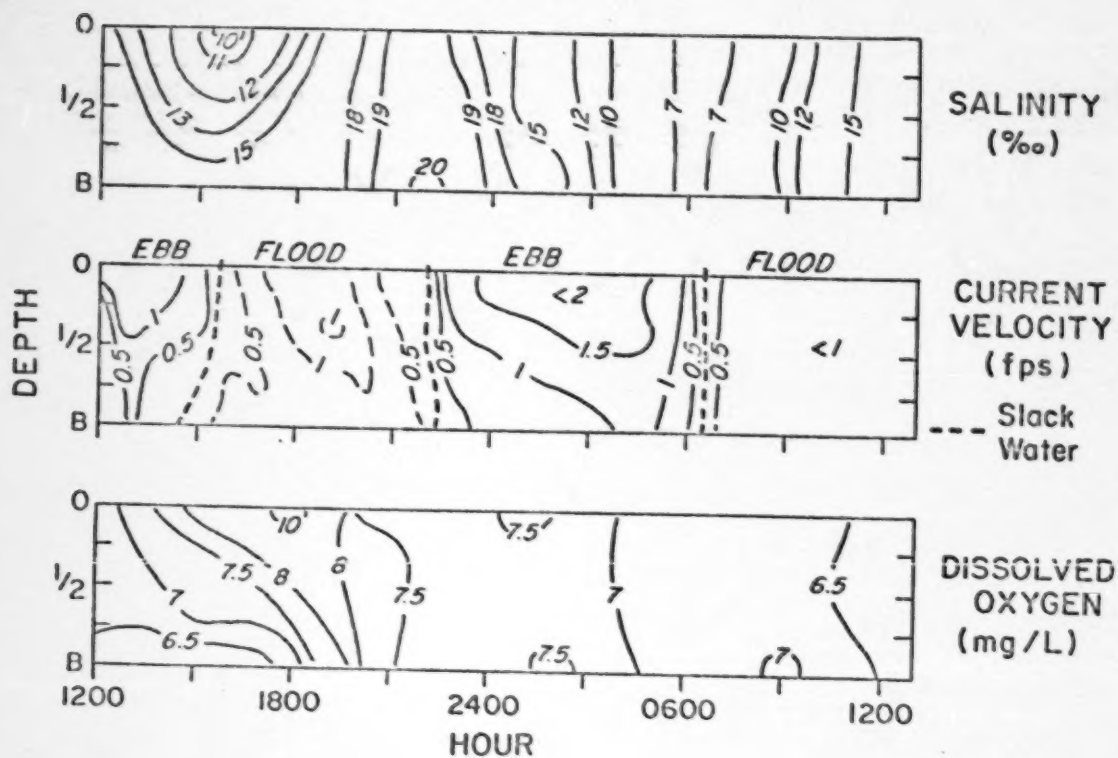
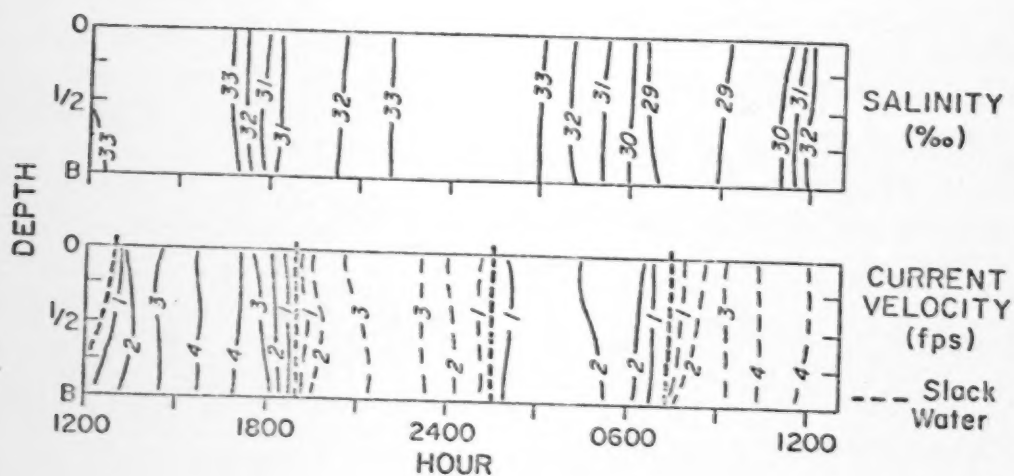


Fig. 1. Estuary type classification according to Hansen and Rattray (1966). See text.



YAQUINA RIVER ESTUARY R.M. 14 AUGUST 1-2, 1967
(EPA DATA, UNPUBLISHED)

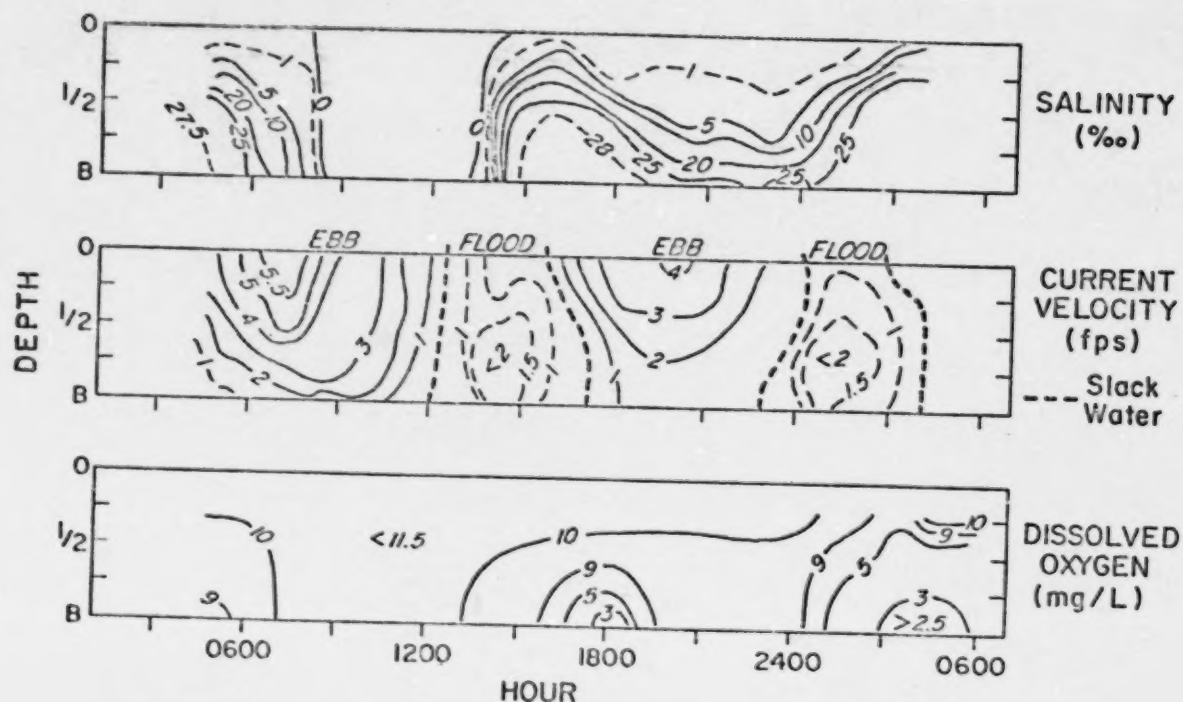


COOS BAY R.M. 8 SEPTEMBER 20-21, 1960
(From W.B. McAlister and J.O. Elanton, 1963)

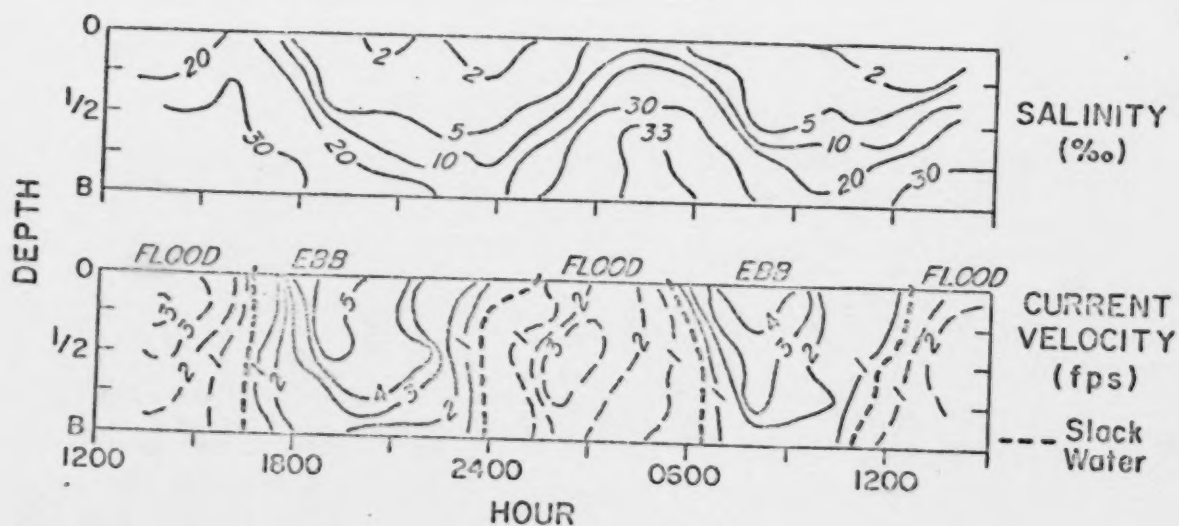
Fig. 2. Time-series of data collected over one tidal cycle.
Fig. 2a-upper, 2b-lower.

periodic stratification of salinity (and oxygen) in the first eight hours followed by well-mixed conditions for the remainder of the time. Oxygen stratification is apparent at the same time but the gradient is directed upward rather than downward as shown at about 1500 hours. If salinity were expressed in terms of the amount of freshwater rather than saltwater, the gradients would be in the same direction; more important, however, is the fact that contours of different substances will rarely be matching. A description of this type of disparity is impossible to rectify by conventional mathematical models and recourse must be made to models that incorporate terms--aside from diffusion--to at least account for photo-synthesis and respiration. The discrepancy is put into perspective by noting that sources of freshwater and oxygen, e.g., are distinctly separate mechanisms. The freshwater source is upriver while atmospheric oxygen can be supplied locally at the surface by reaeration and from within by photosynthesis. The sinks are also different, freshwater decreasing by seepage through the bottom and evaporation while oxygen decreases primarily by within-stream BOD, respiration and bottom demand.

The use of 1-dimensional, longitudinal models implies that the estuary is well-mixed vertically and laterally. (See Ward and Fischer, 1971, for additional comments on the limitations of 1-dimensional models). The Yaquina is well mixed part of the time, but averaged over a tidal cycle it still exhibits stratification. In general, increasing the period of averaging serves to smooth out intra-tidal fluctuations. This rationale has been employed in order to average out diurnal tidal fluctuations in some models, primarily for ease of computation and to keep the problem within computing bounds. The



UMPOUA RIVER ESTUARY R.M. 23 MARCH 21-22, 1961
(Callaway, 1961)



COLUMBIA RIVER R.M. 5 SEPTEMBER 22-23, 1959
(From U.S.A.C.E. DATA: RANGE 2 STATION C, U.S.A.C.E., 1960)

Fig. 3. Time-series of data collected over one tidal cycle.
Fig. 3a-upper, 3b-lower.

of salt in along the bottom to about 28 ppt. The surface speeds associated with buildup of freshwater were in excess of 4-5.5 fps decreasing to 1.5 fps at the bottom during the ebb cycles. The flood speeds were somewhat less with the maximum speeds being located at depth. This was also a period of upwelling; the two flood cycles brought in water low in oxygen at depth, some 7-8 mg/l less than that in the surface. Mathematically modeling this sort of condition would obviously not be a trivial exercise, although two-layer modeling is now being carried out (see, e.g. Vreugdenhil, 1970).

Finally, data for the Columbia River at river mile 5 are shown in Figure 3b. This again typifies high runoff conditions although at this river mile complete flushing does not occur as in the Umpqua example. Similar features are apparent, however, such as the submerged velocity peak on the flood cycle and surface maxima on the ebb. The duration of the ebb cycle is greater than the flood, the difference becoming less disparate with depth i.e., ebb begins earlier in the surface layers and ends later.

In summary, it should be stated that 1-dimensional models of hydraulic phenomenon are rather well developed. Amplitudes and phases of tides may be represented rather faithfully but this in no way ensures that the pollution problems associated with the dispersion of an effluent is reproduced in any such satisfying fashion. This is mainly due to the fact that most non-conservative substances usually show some degree of stratification, especially when surface boundary conditions are in effect and when interactions within the water column and at the sediment-water interface are likely to occur.

TIDES AND CURRENTS AS BOUNDARY CONDITIONS

Tidal information is of interest in the modeling of estuarine pollution problems for a number of reasons: (1) tidal heights or currents are sometimes required as natural boundary conditions; (2) the tide range provides an indication of the amount of energy available for mixing--being proportional to the square of the range; (3) local tidal heights determine the amount of water available for dilution; (4) the excursion of pollutants depends on the range and duration of the ebb and flood tides; (5) in conjunction with (2) above, the vertical salinity profile can be determined in advance given the runoff and tidal prism.

Information on tides in the Yaquina, Alsea, Siletz and Coos estuaries has been presented by Neal (1966), Goodwin, et al. (1970), and Blanton (1970).

Prediction of tidal heights by harmonic analysis is well-established; discrepancies from predicted heights occur due to storm surges and freshets due to local precipitation. Tidal current prediction is less well-behaved from the viewpoint of predictability (as exemplified by ESSA publications); unlike heights they are subject to cross-stream variations and are also markedly affected by winds, local runoff conditions. Both currents and heights are affected by channel modifications.

Prediction of tidal heights and currents becomes less certain with distance upstream as the damping effect of the estuary becomes less dominant. The linearized equations of motion, or the equations where the non-linear terms are neglected altogether, may be quite adequate in the ocean or coastal regions, but definitely cannot be

employed in shallow water cases where topographic effects, friction and the Bernoulli acceleration govern the shape of the wave.

Of the three general methods (the harmonic, characteristic, or numerical method) of solving the equations of tidal motion, the numerical scheme is probably best suited for incorporating variable river flow inputs, winds and irregular topography; the examples given later utilize the numerical scheme.

Tidal Constituents

First of all, consider the tide generating constituents if only to provide a basis for correctly specifying the tide as a boundary condition.

Tidal constituents are the result of trying to account for perturbations in the orbits of the sun and moon. Each constituent represents, in effect, a miniature planet revolving about the earth exerting an attractive force on the earth's water surface. At any given prediction station the constituent is expressed as a periodic sinusoidal elevation of the surface and has an associated amplitude, period and phase; these terms are computed by harmonic analysis of time-series data. The synthesis of the harmonic constituents with reference to a specific datum gives the local tide height; prediction tables usually report the time of high or low water and the water level and the times of slack waters and maximum ebb and flood speeds.

The principal constituents are designated as M_2 , S_2 , N_2 , K_1 , O_1 , P_1 , etc., where the subscript (₁) refers to the diurnal and the (₂) refers to the semi-diurnal nature of the constituent. The relative amplitude of the various constituents determines the type of tide that will occur, i.e., whether it will be semi-diurnal with two nearly equal

highs and lows per day or "mixed" with unequal highs and lows, or "diurnal" with only one high and one low per day.

In Table 3, constituent amplitudes, H, phase angles, K, and mean tide level elevations are given for Oregon-Washington ports^{1/}. Also shown is the ratio $(K_1 + O_1)/(M_2 + S_2)$ ^{2/}; for values of this ratio less than 0.25 the tide is semi-diurnal; between 0.25 and 1.25, it is mixed. It is diurnal for ratios greater than 1.25. It can be seen that the coastal ratios all indicate mixed tides. In the approaches to Puget Sound, however, the ratio indicates diurnal tides as is shown as an example for the Friday Harbor station.

^{1/} Abstracted from U. S. Coast & Geodetic Survey data supplied by LCDR L. Swanson, NOAA.

^{2/} The ratio $(K_1 + O_1)/M_2$ is sometimes used.

TABLE 3
Tidal Constituents - Oregon-Washington Coast

Place and Position	Constant	K ₁	O ₁	P ₁	Q ₁	M ₂	S ₂	N ₂	K ₂	M ₄	M ₆	Z ₀ ft.	$\frac{K_1 + O_1}{M_2 + S_2}$
Crescent City, Ca. 41°45'N 124°12'W	H (ft) k ₀ (deg)	1.276 104	0.785 088	0.386 100	0.145 080	2.334 323	0.579 343	0.500 297	0.145 329	0.006 052	0.008 303	3.7	0.708
Brookings, Or. 42°03'N 124°17'W	H k ₀	1.319 103	0.808 088	0.397 097	0.135 081	2.399 325	0.615 346	0.512 298	0.158 344	0.003 286	0.009 243	3.7	0.706
Port Orford, Or. 42°44'N 124°30'W	H k ₀	1.402 107	0.866 091	0.413 103	0.152 083	2.451 328	0.625 351	0.515 303	0.160 348	0.005 045	0.005 298	4.0	0.737
Bandon, Or. 43°07'N 124°25'W	H k ₀	1.221 112	0.762 098	0.372 109	0.130 095	2.404 335	0.613 000	0.496 310	0.156 351	0.011 244	0.009 299	3.6	0.657
Coos Bay, Or. 43°23'N 124°13'W	H k ₀	1.148 137	0.658 124	0.315 137	0.110 132	2.433 019	0.536 055	0.436 003	0.152 044	0.147 227	0.094 068	4.0	0.608
Gardiner, Or. 43°44'N 124°07'W	H k ₀	1.176 131	0.681 117	0.323 130	0.114 126	2.322 008	0.523 040	0.445 346	0.167 020	0.059 245	0.034 063	3.6	0.653
Florence, Or. 43°58'N 124°06'W	H k ₀	1.162 128	0.648 115	0.312 124	0.101 125	2.220 003	0.517 034	0.422 344	0.155 018	0.033 255	0.024 041	3.5	0.661

TABLE 3 (Cont'd)
Tidal Constituents - Oregon-Washington Coast

Place and Position	Constant	K ₁	O ₁	P ₁	Q ₁	M ₂	S ₂	N ₂	K ₂	M ₄	M ₆	Z ₀ ft	$\frac{K_1 + O_1}{M_2 + S_2}$
Waldport, Or. 41°26'N 124°04'W	H (ft) k ₀ (deg)	1.314 121	0.773 105	0.387 118	0.121 104	2.607 354	0.669 023	0.516 331	0.181 017	0.044 193	0.047 348	4.1	0.637
Newport, Or. 44°38'N 124°03'W	H k ₀	1.386 116	0.843 100	0.426 110	0.141 093	2.778 347	0.728 015	0.566 323	0.190 012	0.044 115	0.021 291	4.3	0.636
Astoria, Or. Youngs Bay 46°10'N 123°51'W	H k ₀	1.326 130	0.798 116	0.346 125	0.130 115	3.129 012	0.746 044	0.586 348	0.216 027	0.054 294	-	4.5	0.548
Raymond, Wn. Willapa R. 46°41'N 123°45'W	H k ₀	1.405 133	0.828 118	0.417 128	0.137 122	3.503 016	0.902 049	0.686 353	0.192 035	0.133 193	0.106 041	5.3	0.507
Toka Point, Wn. Willapa Bay 45°42'N 123°58'W	H k ₀	1.383 127	0.841 111	0.430 122	0.154 105	3.147 006	0.813 036	0.635 340	0.235 024	0.059 196	0.033 034	4.7	0.562
Aberdeen, Wn. 45°58'N 123°51'W	H k ₀	1.336 132	0.790 118	0.394 130	0.145 115	3.418 015	0.856 052	0.672 353	0.252 049	0.168 242	0.134 061	5.6	0.497
Friday Harbor, Wn. Oceano. Lab. 48°33'N 123°00'W	H k ₀	2.463 157	1.413 136	0.768 156	0.231 126	1.827 125	0.423 151	0.394 097	0.114 162	0.063 351	0.010 228	4.6	1.723

STEADY STATE MODELS

Deterministic models of the environment are either steady-state or time-varying, the steady-state assumption implying no time change term in the AD (or hydraulic) equations. The effluent is discharged at a constant rate, and has come into equilibrium with the receiving waters; freshwater inflow and other characteristics of the environment are also steady. The topography of the estuary can be modeled quite closely, e.g., tide level cross-sectional areas can be incorporated to show the irregular nature of the geographic setting. However, the effect of tidal height variations on cross-sectional areas (hence, water volume changes) and tidal current fluctuations cannot be modeled here except by repeated application of the steady-state case, in which case there would evolve a process of simulation. The upstream distribution of an effluent from its source is provided for by a form of exchange coefficient. Simulation of various reaction rates, river flows, diffusion and reaeration rates is a logical extension of the steady-state assumption and perhaps the best justification for its use, since the expected range of concentration of a given pollutant can be easily explored. Input information to a complex area can be obtained from existing hydrographic charts, flows can usually be extracted from Federal or local government publications or files. The actual use of a developed steady-state model, as opposed to the judgment necessary to carry out a realistic simulation, is elementary. Modeling tidal flat areas which are periodically exposed is an exceedingly complex undertaking; properly done it will consist of aerial photogrammetric surveys and precise leveling.

In the highly industrialized setting of the Delaware Estuary the steady-state case has been used as the foundation of a linear programming method of meeting certain water quality standards. For instance, for wastes of known volume, concentration, and reaction rates discharged at various locations along some miles of an estuary and a dissolved oxygen standard of, say, 5 ppm, the linear programming concept can be used in conjunction with the steady-state case to ensure that this goal will be met a given percent of the time and at the least expense to the parties involved. Various external constraints are, of course, involved here, but the tools are available for the exercise of logical and unarbitrary decision making.

Two steady-state models are described below in order to exhibit some of the uses of this approach to estuarine problems. Both models were applied to Grays Harbor.

Grays Harbor, Washington

The Grays Harbor entrance is about two miles in width, but shoals extending southward from Point Brown contract the navigable channel to a width of 0.4 miles (Fig. 4). From the entrance, the bay extends eastward for 15 miles to the mouth of the Chehalis River. The bay is filled by shoals and flats which are bare at low water and cut by numerous channels.

The mean range of tide at Aberdeen is 7.8 feet. The range between mean lower low water and mean higher high water is 9.9 feet and a spring range of about 14 feet occurs. The flushing rate at 700 cfs was calculated by the tidal excursion-prism method to be 5.5 days.

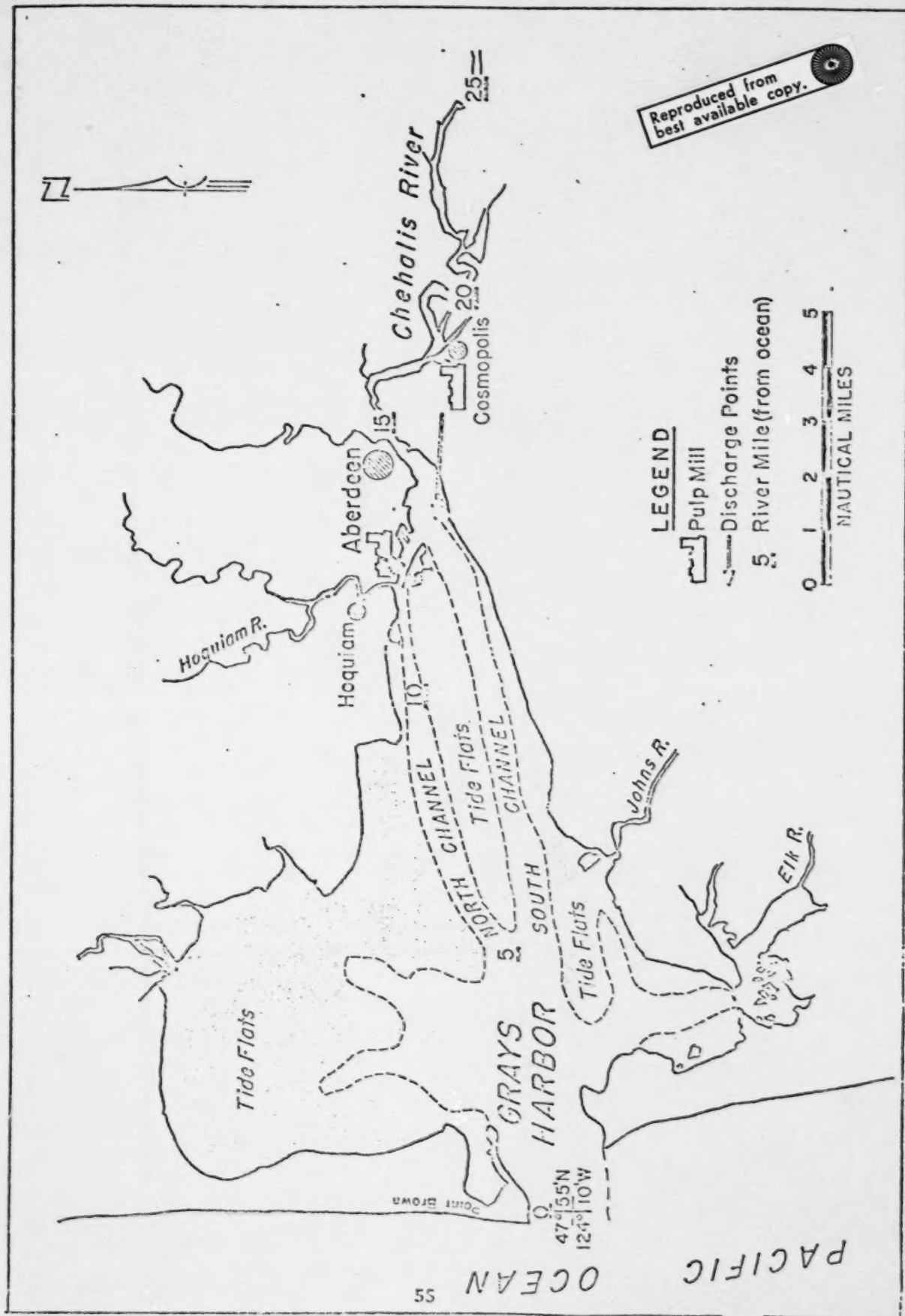


Figure 4. Location map, Grays Harbor, Washington.

Eriksen and Townsend (1940) have presented data available for calculation of eddy diffusivities from the longitudinal salinity profile in the harbor. During a period of a constant flow of 700 cfs well-mixed, essentially 1-dimensional, conditions existed.

The influence of the extensive tide flat areas on the distribution of waste effluents is unknown, and the computational error introduced by their presence cannot be evaluated at this time. (Dr. Bella's paper on the influence of tidal flats on water quality should provide some insight on this subject.) The tide flats were tacitly included in the model by using cross-sectional areas based on mean tide level.

Two pulp mills in the area discharge partially treated wastes directly to the estuary or to holding lagoons for discharge at high flow periods. The discharge of oxygen-demanding wastes naturally contributes to the oxygen deficit in addition to bottom demands and plant respiration. Another factor which enters and which frequently occurs on the West Coast of the United States is the process of upwelling. During periods of sustained northerly winds, surface waters are transported away from the coast and replaced by waters at depth. The dissolved oxygen content of the deeper waters may be as low as 1 to 2 mg/l and have been observed as penetrating into the harbor (Pearson & Holt, 1960). Prevailing winds that induce upwelling generally occur during the low river runoff period and can compound the effect of low dissolved oxygen concentrations resulting from estuarine waste discharge practices.

When only one industrial or municipal sewage facility affects an environment, the problem of a desirable level of waste treatment is fairly straightforward if water quality objectives are stated. When two or more facilities discharge into the same system, then an amenable

treatment level requires that the contribution of each facility to the total stream waste load be capable of isolation. Examination of field data alone cannot separate the concentration distribution observed in the waterway into individual facility components, since the concentration at any point depends not only on the amount but also upon the location of waste discharge. Upwelling compounds the situation.

Stommel's Model

A model for treating the distribution of a pollutant in a well-mixed, 1-dimensional tidal estuary was given by Stommel (1953) as:

$$(4) \quad F(x) = QL - SA \frac{dL}{dx}$$

where $F(x)$ = net seaward flux of pollutant across a section

A = turbulent eddy diffusivity ($L^2 T^{-1}$)

Q = river flow ($L^3 T^{-1}$)

L = concentration of substance (M/M)

S = channel cross-sectional area (L^2)

The steady-state equation (4) for a non-conservative pollutant may be written as:

$$(5a) \quad \frac{d}{dx} (QL - SA \frac{dL}{dx}) + K_1 SL = 0$$

or

$$(5b) \quad \frac{d}{dx} (QL - SA \frac{dL}{dx}) + K_1 SL = J$$

at an outfall where J is the rate of flow of pollutant (MT^{-1}) at the outfall, and the reaction term, $K_1(T^{-1})$, is taken as a constant throughout

the estuary. Burt and Marriage (1957) employed equations (5) in a study of the potential pollution of Yaquina Bay, Oregon.

Stommel's relaxation solution of n linear differential equations was replaced by a matrix algebra solution by Callaway (1966) which was first employed by Dorrestein (1960) and later extended to the time-variable case by Thomann (1963). After expressing equations (5) in finite difference form they become, utilizing matrix notation:

$$(6) \quad [A] [L] = [J]$$

where $[A]$ is an $n \times n$ matrix which includes the advection, diffusion and decay terms, $[L]$ is a column vector of dimension n representing BOD concentration and $[J]$ is a column vector of dimension n representing forcing functions of waste discharges. Equation (6) is easily solved by matrix inversion on a digital computer:

$$(7) \quad [L] = [A]^{-1} [J]$$

The output $[A]^{-1}$, which is the inverse of $[A]$, is multiplied by $[J]$ in a given section where an outfall is situated. If outfalls are located along the estuary the resulting outputs may be added to give the total longitudinal output since the system is linear and superposition of solutions applies.

Longitudinal DO Profile

The computation of the steady-state DO profile is somewhat more complex than that for describing the distribution of a conservative or non-conservative pollutant and was not treated by Stommel. The basic dissolved oxygen equation, avoiding finite difference formulation and neglecting photosynthesis and sludge bed demand, is:

$$(8) \quad \frac{d}{dx} (QC - SA \frac{dC}{dx}) = K_2 S(C_s - C) - K_1 SL$$

where C = dissolved oxygen concentration (ML^{-3})

C_s = saturation concentration (ML^{-3})

K_2 = reaeration coefficient (T^{-1})

and the other terms are as before. Modifying equation (5a) for the distribution of pollutant or BOD:

$$(9) \quad \frac{d}{dx} (QL - SA \frac{dL}{dx}) + K_1 SL = F$$

where F is the forcing function.

Upon differentiating equation (8) with respect to x and rearranging, we have:

$$(10) \quad [Q \frac{d}{dx} - \frac{d}{dx} (SA) \frac{d}{dx} - SA \frac{d^2}{dx^2} + K_2 S] C = K_2 SC_s - K_1 SL$$

for dissolved oxygen distribution, where L is determined from (11) as:

$$(11) \quad [Q \frac{d}{dx} - \frac{d}{dx} (SA) \frac{d}{dx} - SA \frac{d^2}{dx^2} + K_1 S] L = F$$

In equation (10), the terms in brackets represent a system of operations, or transfer function, which transforms the forcing functions $K_2 SC_s$ and $K_1 SL$ into an output C . Similarly, the forcing function F in equation (11) is the BOD discharged at an outfall section somewhere in the estuary which is transformed by the advection, variable diffusion and constant reaction terms within brackets to a longitudinal BOD distribution. At each end of the model $C = C_s$, and eddy diffusivity at the river (freshwater) end is taken as zero. Although Stommel's model is applicable to tidal freshwater streams, the evaluation of eddy diffusivity would have to be

made through the use of dye releases or some other substance beside salinity as used here.

Writing equation (10) in matrix notation for the system of equations:

$$(12) \quad [B] [C] = [K_2 SC_s - K_1 SL]$$

and solving for [C]:

$$(13) \quad [C] = [B]^{-1} [K_2 SC_s] - [B]^{-1} [K_1 SL]$$

In equation (11) when $F = 0$, $L = 0$ and equation (8) reduces to $C = C_s$ since it is assumed that there is no longitudinal variation in C_s (first and second derivatives of C with respect to x are zero). Then equation (B) is written as:

$$(14) \quad [C] = [C_s] - [B]^{-1} [K_1 SL]$$

Solving for [L] in equation (11) and substituting in (14):

$$(15) \quad [C] = [C_s] - [B]^{-1} [A]^{-1} [K_1 F]$$

which is the synthesized DO profile equation.

Twenty-three sections were used in the 700 cfs model, no attempt was made to assess temperature effects on the BOD rate constant: C_s ($= 7.5\text{mg/l}$) was taken as constant in the estuary. (The forcing functions used in the above derivations have awkward dimensions which can be resolved by dividing the equation by the cross-sectional area term.)

Salinity Profile

Figure 5 shows the computed and observed longitudinal salinity distribution in the estuary at 700 cfs. Computed results were taken from the matrix inversion of the system represented by equations (5) with $K_1 = 0$. Observed salinity at the ocean end of the model was taken as the input forcing function.

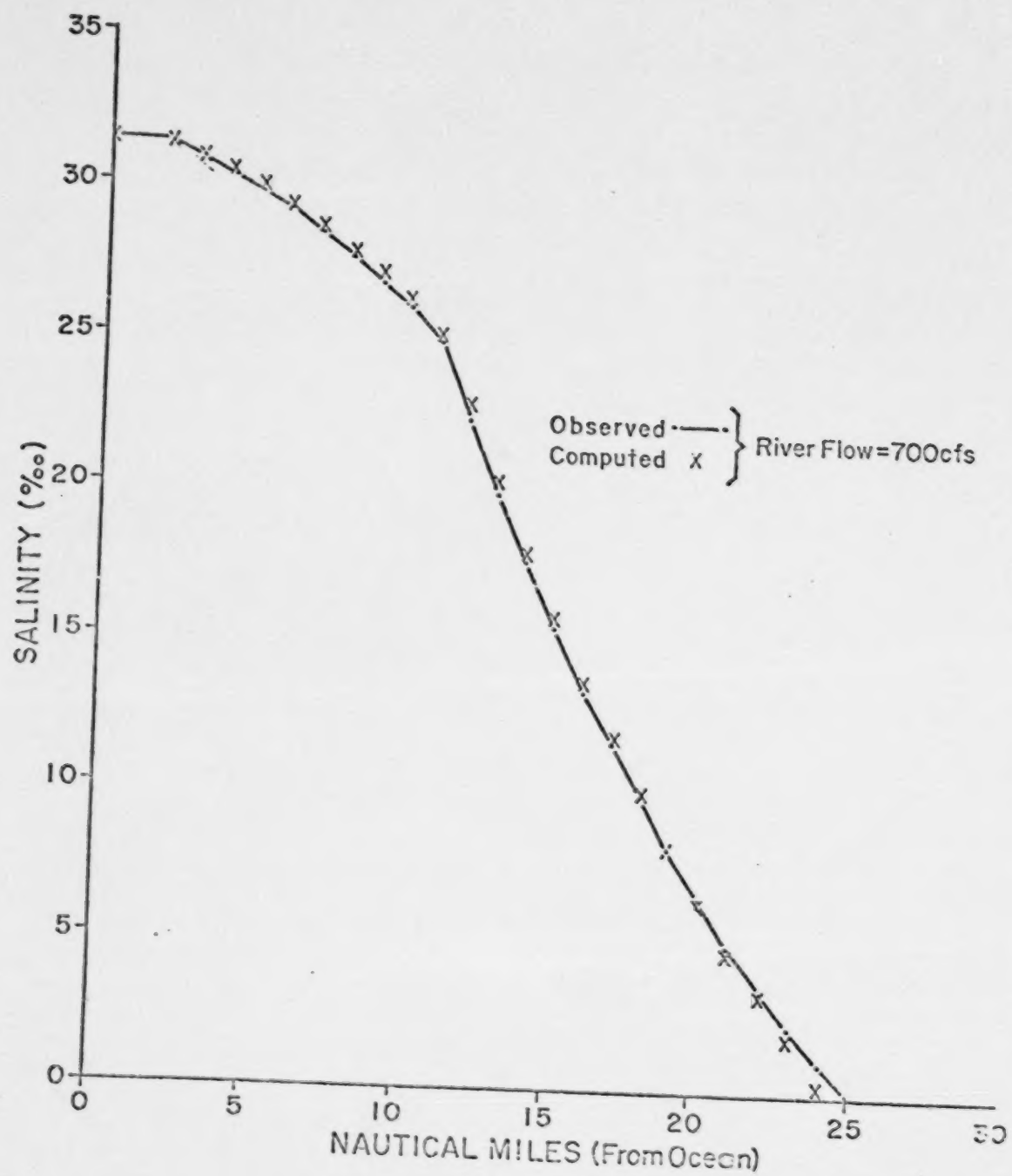


Fig. 5. Salinity profiles, Gray's Harbor.

was taken as the input forcing function. The "computed" salinity serves mainly as a check on the calculation of the coefficients in (9) since the observed salinity data were used in the calculation of eddy diffusivities. Diffusivities calculated for $Q = 700$ cfs averaged 2.6 square miles per day.

Upwelling

At the ocean end of the model upwelling is introduced as an initial oxygen deficit ($C_s - C$) subject to reaeration only. Work by Sverdrup and Fleming (1941) and Barnes and Collias (1958) suggests that waters at depth in the ocean and in fjords have very slow deoxygenation rates. Barnes and Collias find average rates of 0.016 ml/l/day in the fjord-like waters of Puget Sound while Sverdrup and Fleming reported 0.005 ml/l/day at a depth of 200 meters off the California coast. In estimating the upwelling contribution to low DO conditions, then, K_1 has been taken as negligible.

Figure 6 shows an initial oxygen deficit of 5.5 mg/l in the ocean subject to reaeration rates of 0.15 and 0.30 per day within the estuary. In the absence of a pollution source in the estuary the deficit would decrease upstream (C would approach C_s , shown by the dashed line). The upwelling effect for $K_2 = 0.15$ /day is superimposed on an observed DO profile (solid line, Eriksen and Townsend, op.cit.). The dash-dot line shows the superposition of the two deficits; during periods of upwelling, interpretation of field data extending only from mile 8 upstream would attribute the entire oxygen sag to the BOD sources located in the middle of the estuary if the possibility of an upwelling effect were not recognized.

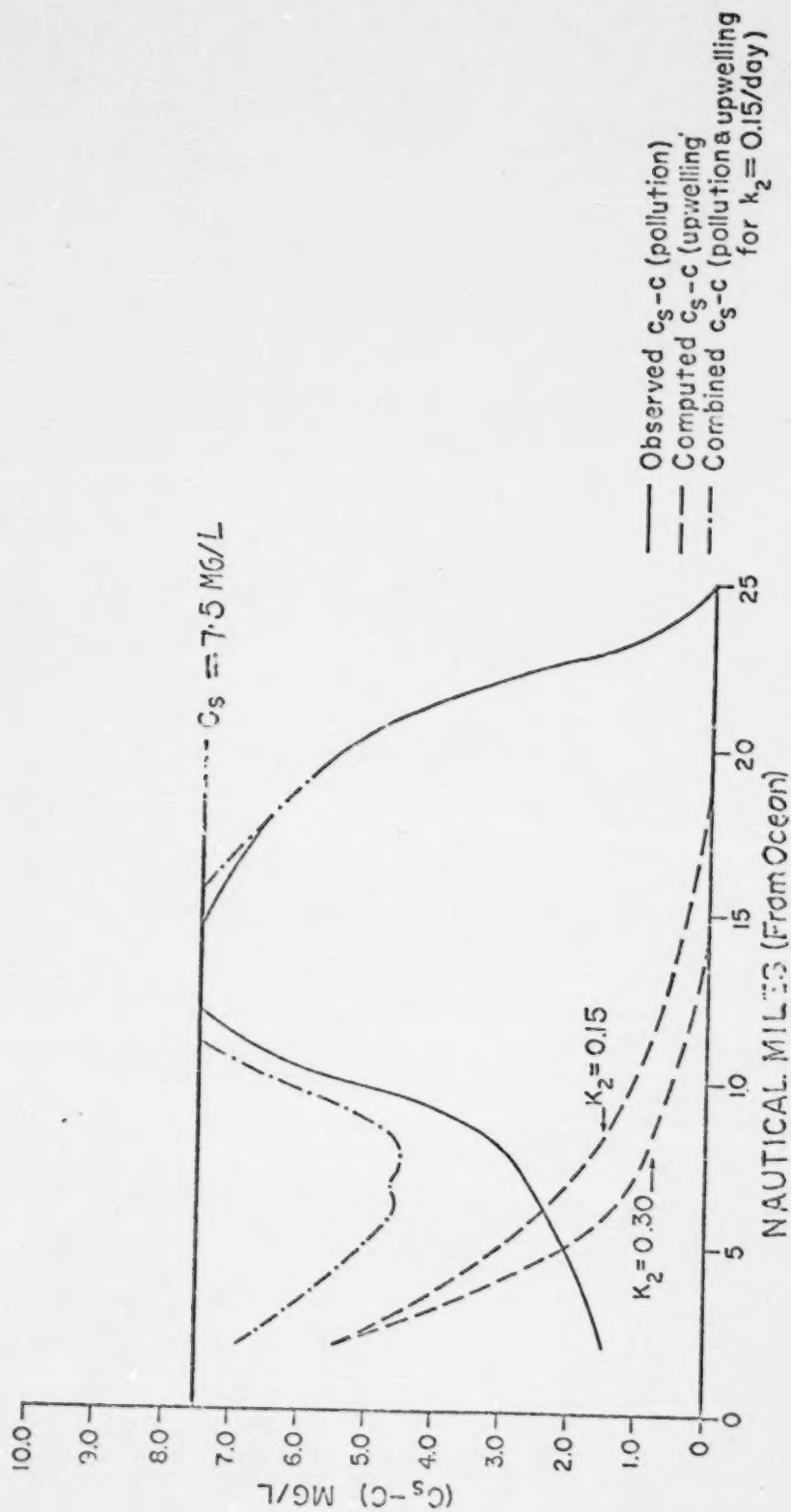


Figure 6. Oxygen deficits ($C_s - C$) resulting from pollution (observed), upwelling (calculated) and pollution plus upwelling.

Thomann's Model

The Thomann model (Thomann, 1963, Thomann and Sobel, 1964) also goes by the name of DECS III (for Delaware Estuary Comprehensive Study); it has been extensively documented by Jeglic (1967).

Some features of the model are similar to Stommel's, but there are enough differences to warrant discussion before the results are presented.

The time variable case is explained below; the examples shown, however, are steady-state.

In matrix form the differential equations for BOD (L) and dissolved oxygen (C) are:

$$\begin{aligned} [B] [L] + \left[\frac{dL}{dt} \right] &= [J] - [L_b] \\ (17) \quad [A] [C] + \left[\frac{dC}{dt} \right] &= [r] [C_s] + [F] - [D] [L] - [C_b] \end{aligned}$$

where at any given time j the matrices $[A]$ and $[B]$ are square, tridiagonal matrices of order n , the nonzero coefficients of which are defined as:

$$A_{i, i-1} = B_{i, i-1} = \frac{\alpha_{i, j} Q_{i, j} + E_{i, j}}{V_{i, j}} \quad [i = 2, 3, 4, \dots, n]$$

$$A_{i, i} = \frac{\phi_{i, j} Q_{i, j} - \alpha_{i+1, j} Q_{i+1, j} - E_{i, j} - E_{i+1, j}}{V_{i, j}} - r_j \quad [i = 1, 2, 3, \dots, n]$$

$$B_{i, i} = \frac{\phi_{i, j} Q_{i, j} - \alpha_{i+1, j} Q_{i+1, j} - E_{i, j} - E_{i+1, j}}{V_{i, j}} - d_j \quad [i = 1, 2, 3, \dots, n]$$

$$A_{i, i+1} = B_{i, i+1} = \frac{E_{i+1, j} - \phi_{i+1, j} Q_{i+1, j}}{V_{i, j}} \quad [i = 1, 2, 3, \dots, (n-1)]$$

where the i subscript denotes the i^{th} interval in a 1-dimensional grid (n grids) and the j subscript represents a time interval. The other variables are as below:

j_i = waste load to segment i

r_j = reaeration coefficient

d_j = decay coefficient

P_j = other sources and sinks of oxygen

V_i = volume of segment i

Q_j = net flow

$E_{i,j}$ = eddy exchange coefficient

α_i, ϕ_i = advection factors

The exchange coefficient is:

$$E_{i,j} = \frac{K_{i,j} \cdot A_{i,j}}{\frac{1}{2} (L_i + L_{i+1})}$$

where $K_{i,j}$ = eddy diffusion coefficient

$A_{i,j}$ = cross-sectional area

L_i = segment length

The advection factors are:

$$\phi_{i,j} = \frac{1}{2} \left(\frac{E_{i-1,j}}{Q_{i-1,j}} + \frac{E_{i,j}}{Q_{i,j}} \right)$$

$$\text{If } \frac{E_{i-1,j}}{Q_{i-1,j}} > \frac{E_{i,j}}{Q_{i,j}} \text{ then } \phi_{i,j} = \frac{E_{i,j}}{Q_{i,j}}$$

If $\phi_{i,j} \geq 0.5$ then $\phi_{i,j} = 0.5$, and

$$\alpha_{i,j} = 1 - \phi_{i,j}$$

The role of the advection factor is similar to the weighting function (discussed, e.g., in Smith, 1965, p.23). The implicit method of solution ensures stability (but not necessarily accuracy); the advection factors minimize numerical errors which resemble diffusion.

The time variable Thomann model does not solve for short-term tidal fluctuations, hence, upstream dispersion is accomplished through the exchange coefficient and not through advection.

Steady-state solutions of equation (17) are shown in Figure 7 as the oxygen deficit per 100,000 pounds BOD discharged at various points in the estuary under various specified conditions. Figure 7a shows the effect of adding 100 cfs increments of flow to the head of the estuary with eddy diffusion calculated from the salinity profile. As can be seen the peak concentration is reduced and moves further downstream. The deficit at sections 14 and above, however, are slightly greater as a result of the increased flow. Figure 7b shows the effect of placing the outfall at different points in the north channel with a 550 cfs flow and specified diffusion rate.

Figures 7c and 7d show the effect of a doubling of the reaeration coefficient with the same decay coefficient in each case. The areas under the individual curves for a release point are greatly reduced for a reaeration coefficient increase from 0.15 to 0.30 per day.

Because the profiles have not been verified the above should be considered as examples of numerical experimentation rather than of actual conditions. A very important (and difficult) part of modeling lies in verification of the model output. Verification, or adjustment, is largely a matter of judgment; unfortunately there is little agreement or precedent upon which to go except in the simplest of cases in which analytical solutions can be simulated.

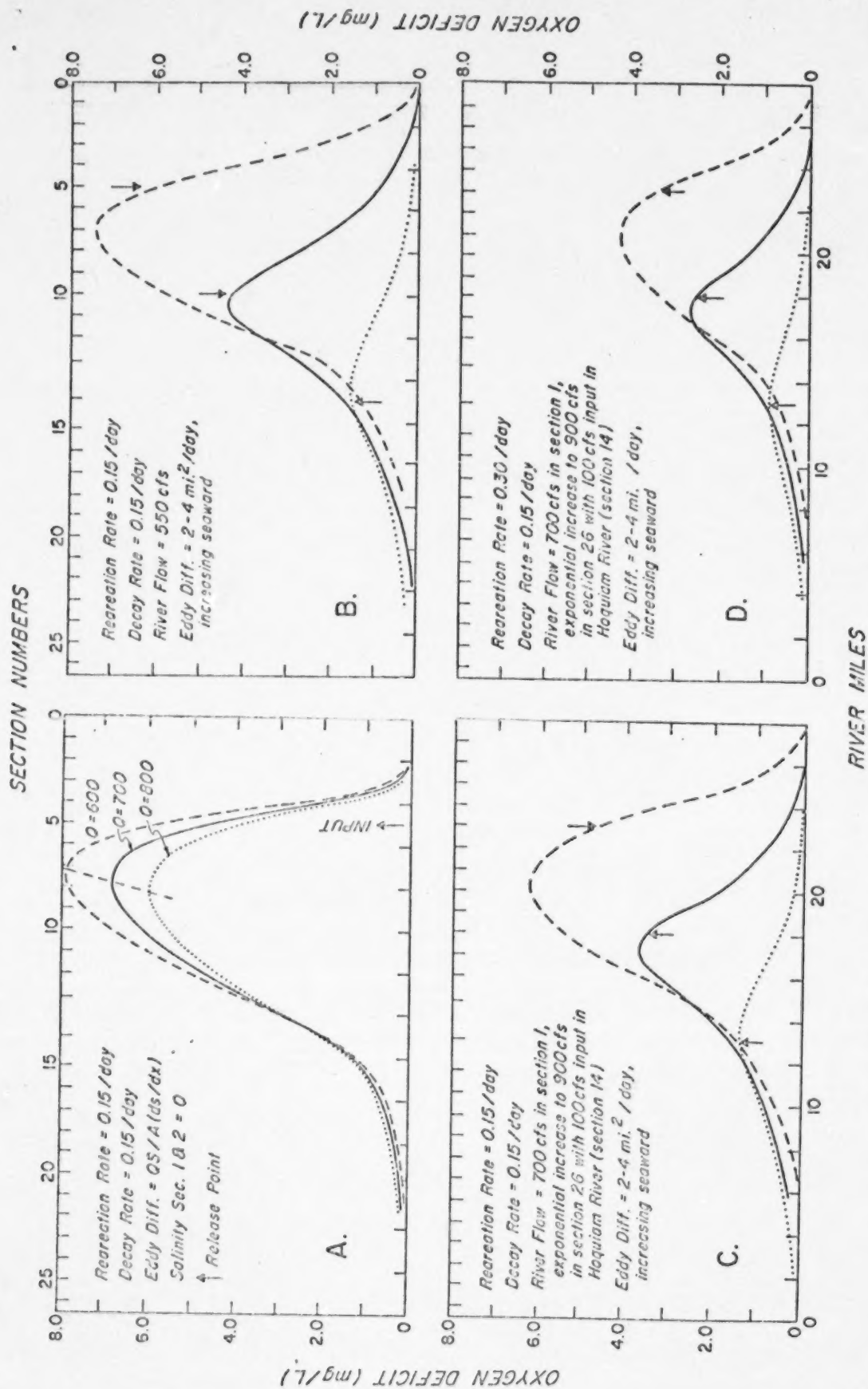


Fig. 7. Computed oxygen deficits, Gray's Harbor, per 100,000 lbs BOD.

TIME VARIABLE MODELS

Columbia River Tides and Currents

The mathematical methods for predicting tides and currents in the Columbia River from the Pacific Ocean to Bonneville Dam have been presented by Callaway, et al. (1969). Briefly, an explicit solution of the 1-dimensional equations of motion and continuity was utilized in a finite element formulation of the geometry of the system; the algorithm used is due to Shubinski and Sheffey (1965). The input requirements for the model are channel lengths, widths, cross-sectional areas, and frictional resistance coefficients, and junction surface areas. Boundary conditions consist of the time varying tidal heights which can be expressed as a Fourier series or equally spaced points and tributary and mainstream river flow data. The model also treats conservative and non-conservative substances and temperature with a surface boundary exchange.

The next few paragraphs deal with verification attempts (Callaway and Byram, 1971); by this is meant duplication (primarily by adjustment of friction) of predicted heights and currents. We were not overwhelmed with actual field data and relied mainly on USC&GS tide tables for verification of the hydraulics. Stage elevations at Bonneville Dam were taken from a report by Bonneville Power Administration.

At the upper boundary condition the flow from the junction representing Bonneville Dam was input. Water elevation above datum was then approached as the channel depths responded to the amount of discharge. At the outset of this study, we were not sure just what the elevations should be and it was pleasurable to find that the computed elevations fell

within 1 to 2 inches of observed, even when the difference in tailwater elevations for two flows was about seven feet.

In Figure 8, computed and observed co-range lines are given. Also shown in the figures are "observed" ranges as taken from the C&GS tide tables. In general, the agreement between computed and observed values is excellent.

Figure 9 shows a longitudinal section of computed and observed ranges. Note the agreement in the lower river which shows an increase in mean diurnal range with distance upstream, passing through 6 feet about mile 5 and again at mile 25. The agreement is rather good along the entire river stretch with the exception of from about mile 70 to 100. The reason for this discrepancy is not known, but could be due to channel modifications made after the USC&GS computations base data were collected but which are incorporated on the charts from which the input data was taken.

The 1959 flow data were chosen as representative of a "normal" low flow period. Starting at river mile 27, a wave with 6 foot range, Manning coefficient of $0.02 \text{ ft}^{1/6}$ and a 12.5 hour period was input at a river level of 3.8 feet (as determined from Corps of Engineers data for the flows used). It was assumed that 25 hours would be sufficient for convergence, hence these output data (first 25 hours) were discarded. Figure 10 shows the progression of the wave with distance upstream. Agreement between computed and Tide Table phase lags and amplitudes was very good.

Clark and Snyder (1969) published a note on their observations of current reversal 70 miles upstream from the mouth of the Columbia

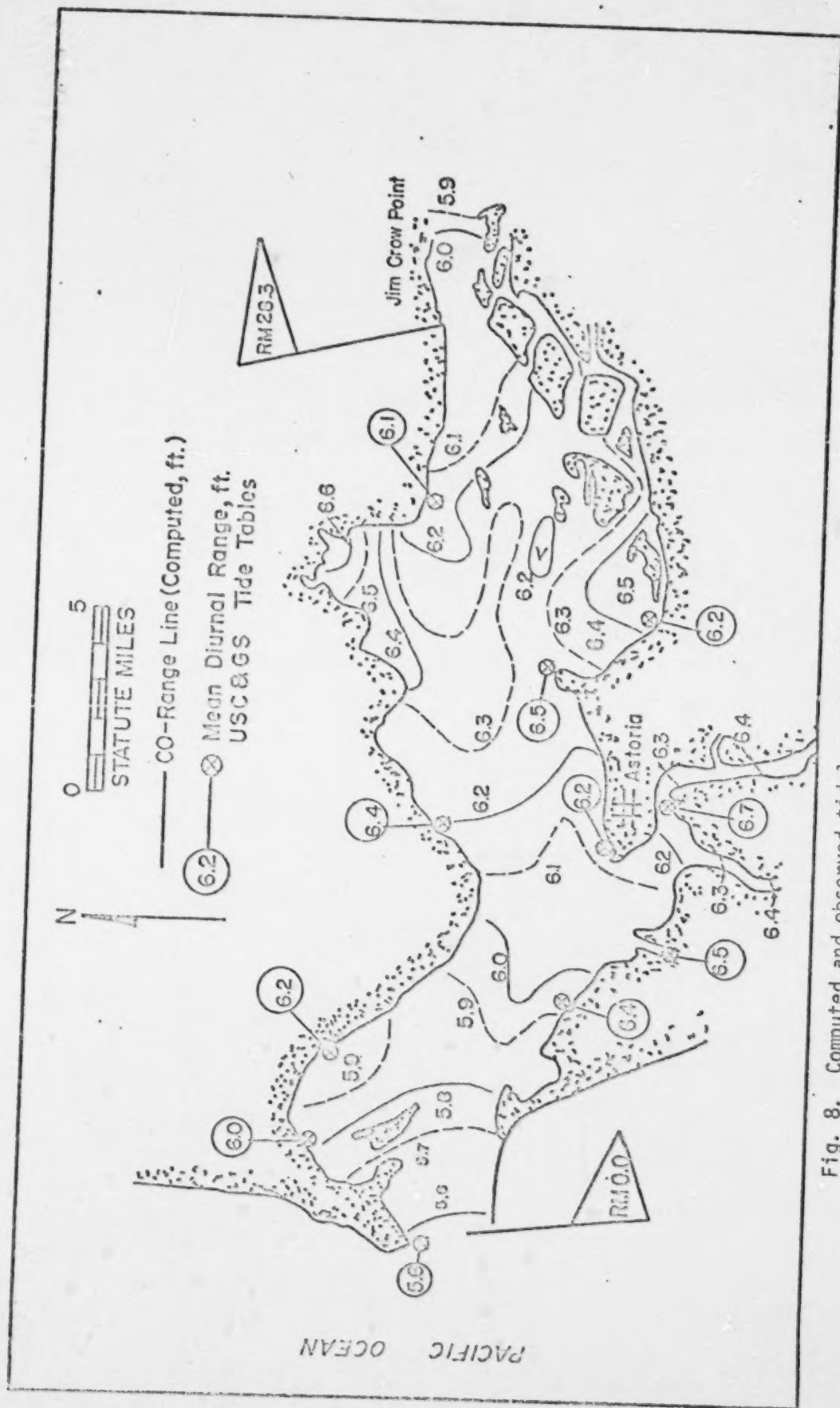


Fig. 8. Computed and observed tidal ranges, Columbia River, miles 0-30.
From Callaway and Byram (1971).

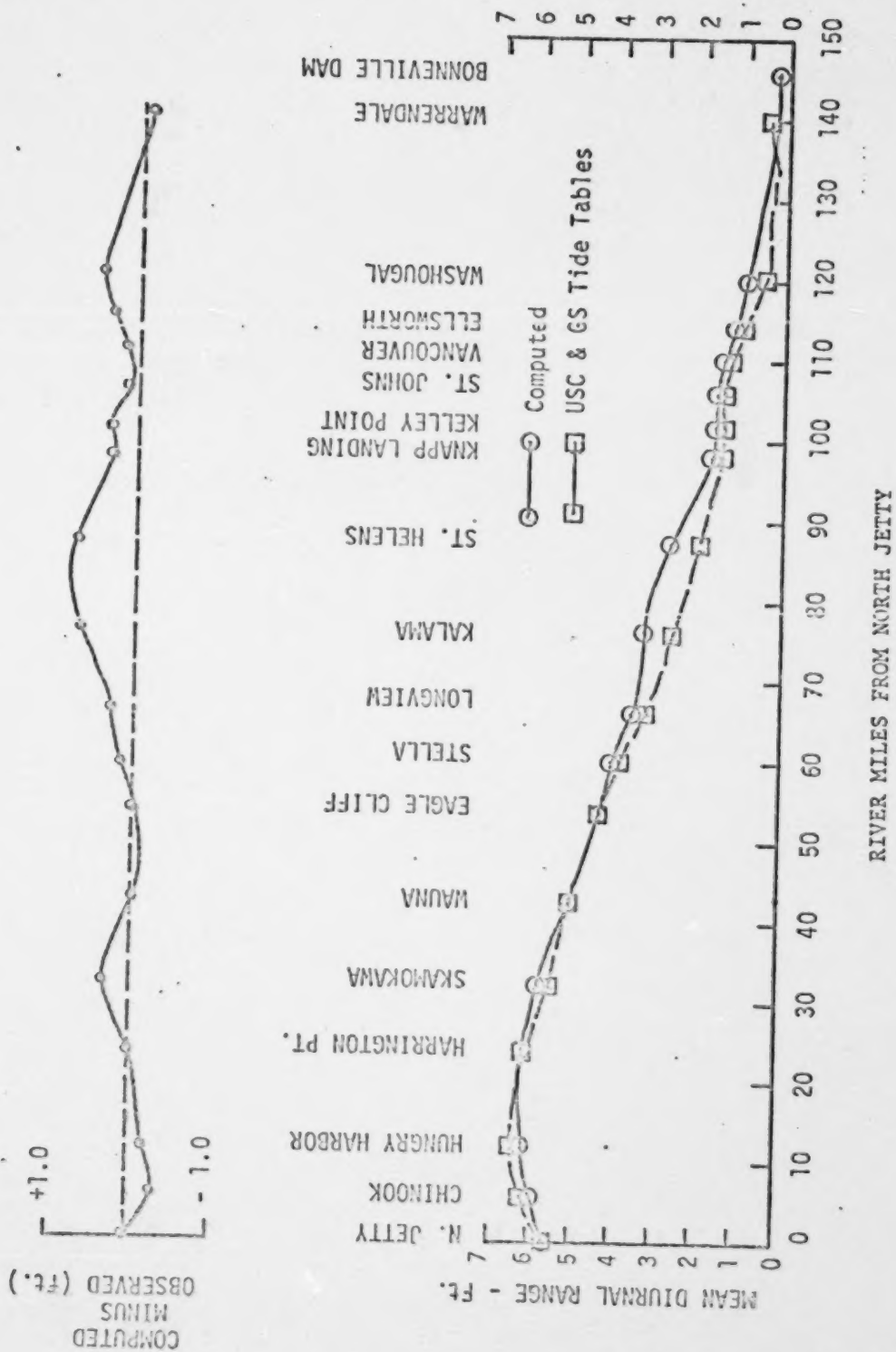


Fig. 9. Computed and observed tidal ranges, mid-channel Columbia River, Pacific Ocean to Bonneville Dam. From Callaway and Wyram (1971).

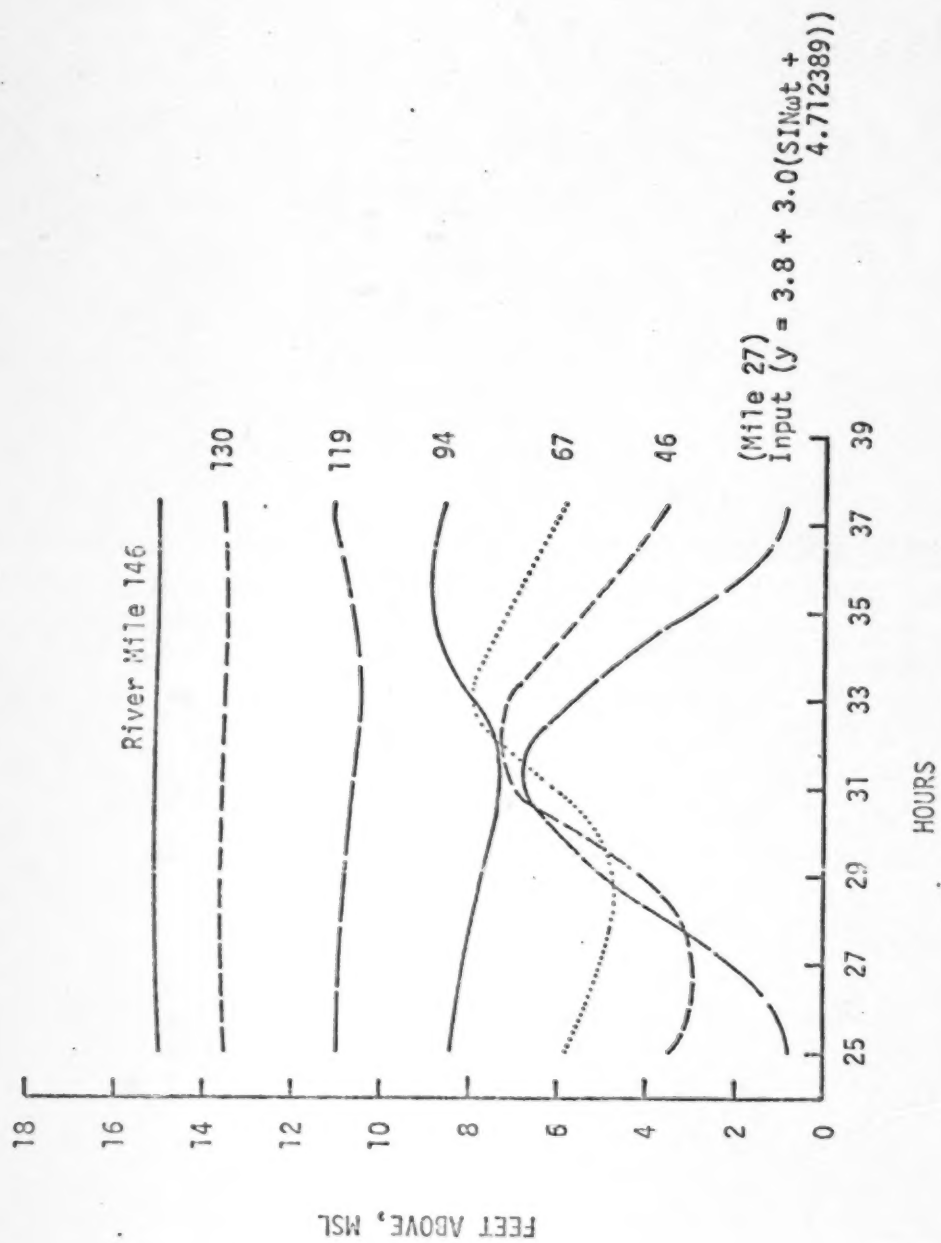


Fig. 10. Computed stage elevations versus time, mile 27-146.
From Callaway and Byram (1971).

River, near the proposed site of two thermal electric power plants. The particular river stretch was chosen because heated cooling water returned to the stream might interfere with the endemic and anadromous fishery, especially if there were to be periods of low current speeds as during a current reversal. The purpose of their study was thus to determine if reversal occurred, and if so, to measure its duration during a very low flow period (84,500 cfs at Astoria) of less than half the river's mean annual discharge. The low flow occurring during their study was the result of filling the then just-completed John Day Reservoir. A rather extensive field study using dye dumps and fluorometry was reported; in summary, they found that reversal occurred over a four-hour period from about 3:50 a.m. to 8:00 a.m. on April 16-17, 1968.

A sine wave with a period of 12.5 hours and range of 7.5 feet, the diurnal range at the North Jetty, was input at the ocean end of the model. A Manning coefficient of $0.02 \text{ ft}^{1/6}$ was employed and conditions simulated for two tidal cycles.

Figure 11 is a computer plot of tidal elevations, velocities and flows at river miles 67 to 74. The somewhat erratic traces in the first few hours of simulation are due to convergence of the numerical solution from the given set of initial conditions. In this case, convergence was complete in about 10 hours. The open circles on the figure are tidal elevations as taken from Clark and Snyder and adjusted so that the computed second high water maximum is in phase with the observed.

Even though no attempt was made to input the existing mixed tide at the entrance (although it could easily have been at the expense of more computer time) the computed curve is in good agreement with the observed.

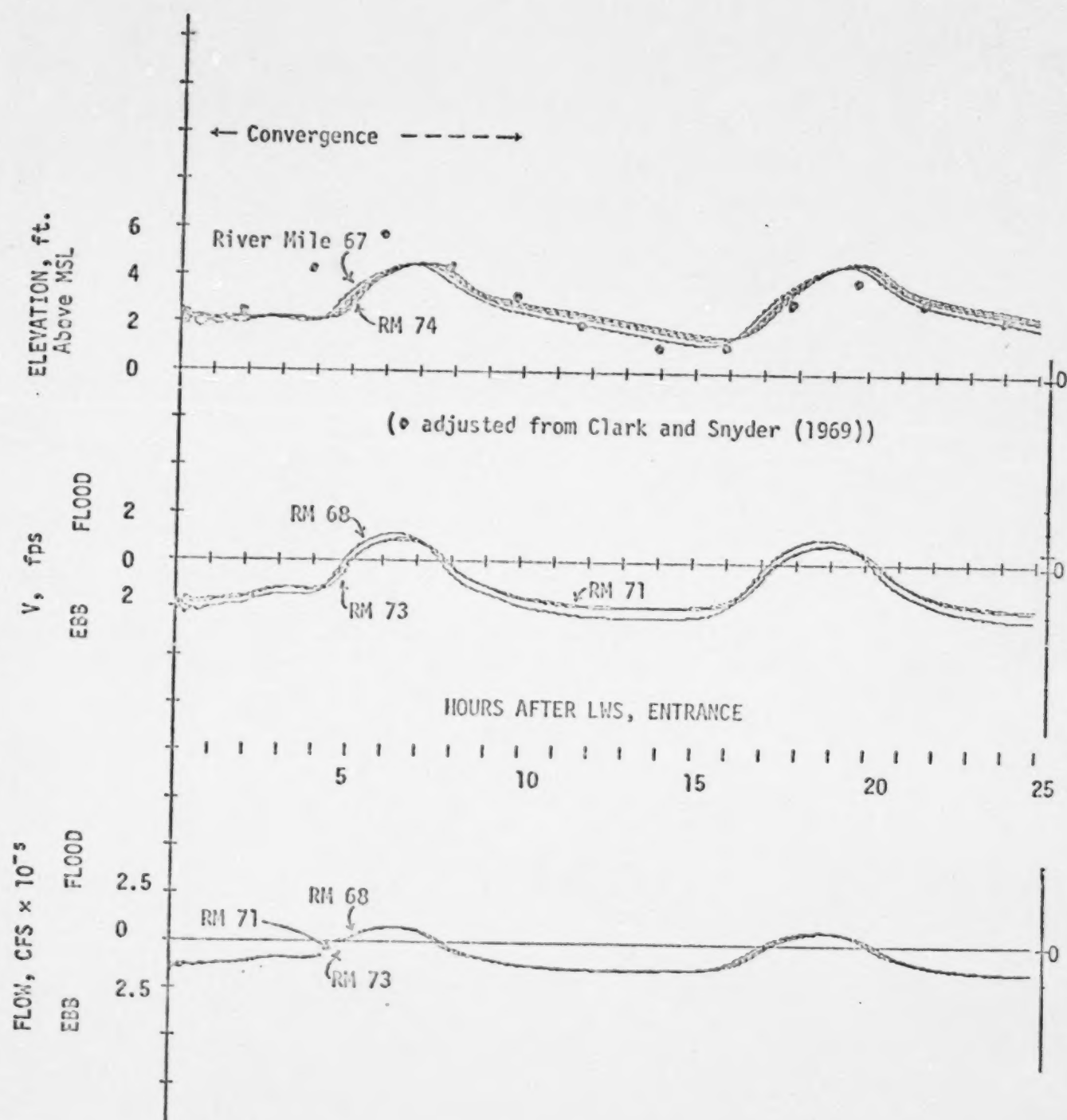


Fig. 11. Computed tidal elevation, current velocity and flow. Columbia River at about river mile 70 for low flow conditions. From Callaway and Byram (1971).

It can be seen that the frictional and non-linear term in the equation of motion have distorted the ocean input sine wave to the output shown near Prescott. Computed tide reversal (middle figure) occurs for about three hours as compared with four hours observed, the difference due mainly because a uniform friction coefficient was used in the model. Computed maximum flood velocity was 1.2 fps; maximum ebb velocity was 2.1 fps. These velocities are integrated vertically across channel and will generally be lower than observed peak velocities in mid-channel.

Computed transport through the channels is shown in the lower part of the figure. New flow in the Columbia channels for the last 12.5 hours of the run was 78,606 cfs; the remaining flow being diverted north through another channel. For the low flow period of 1968, Figures 12 and 13 show computer plots of head, velocity, and volume of flow with river mile upstream for the tidal cycles. These results have not been "verified" as such but are given here as additional examples of output.

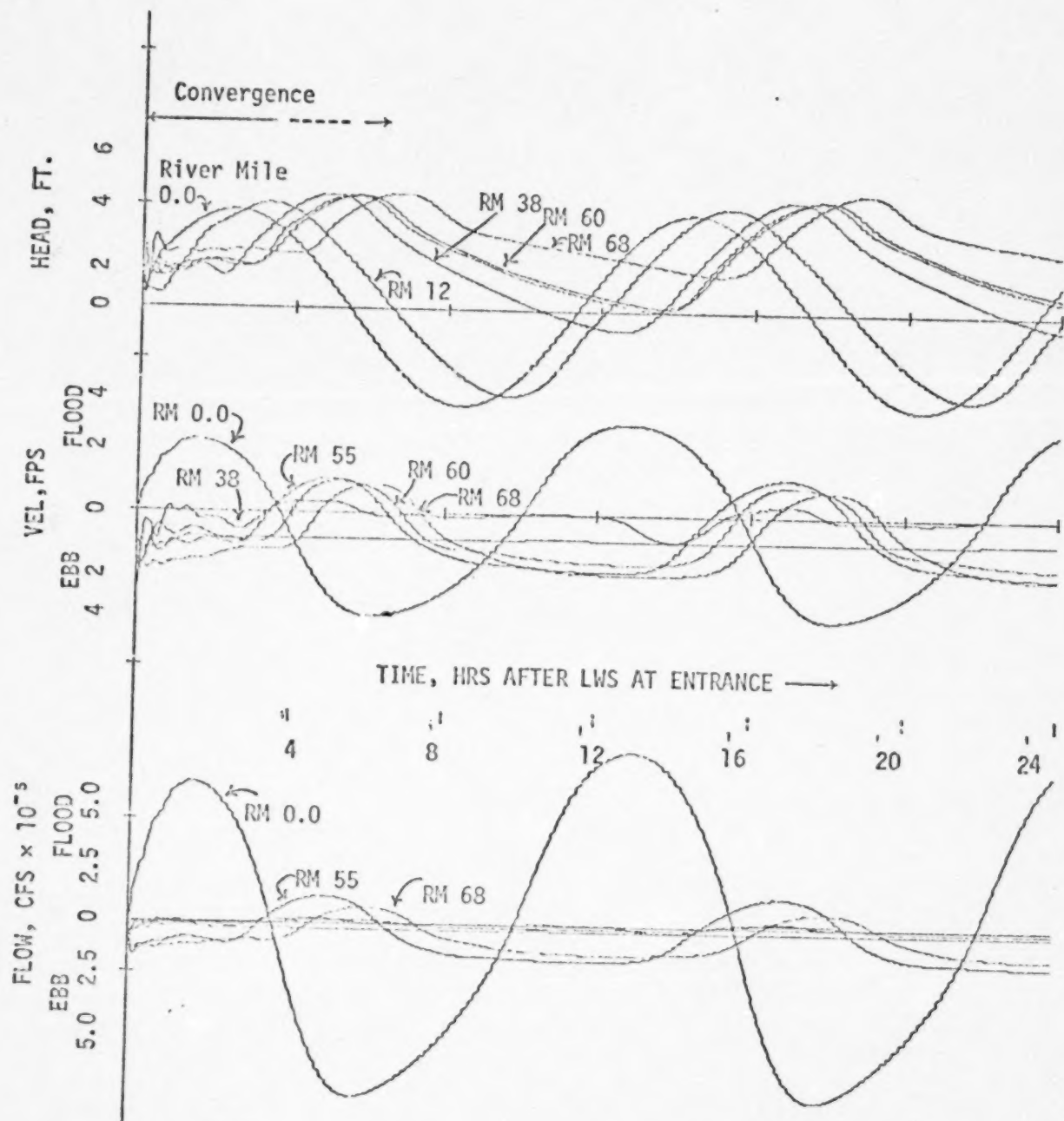


Fig. 12. Computed tidal elevation, velocity and flow. River mile 0-68, Columbia River, low flow conditions. From Callaway and Byram (1971).

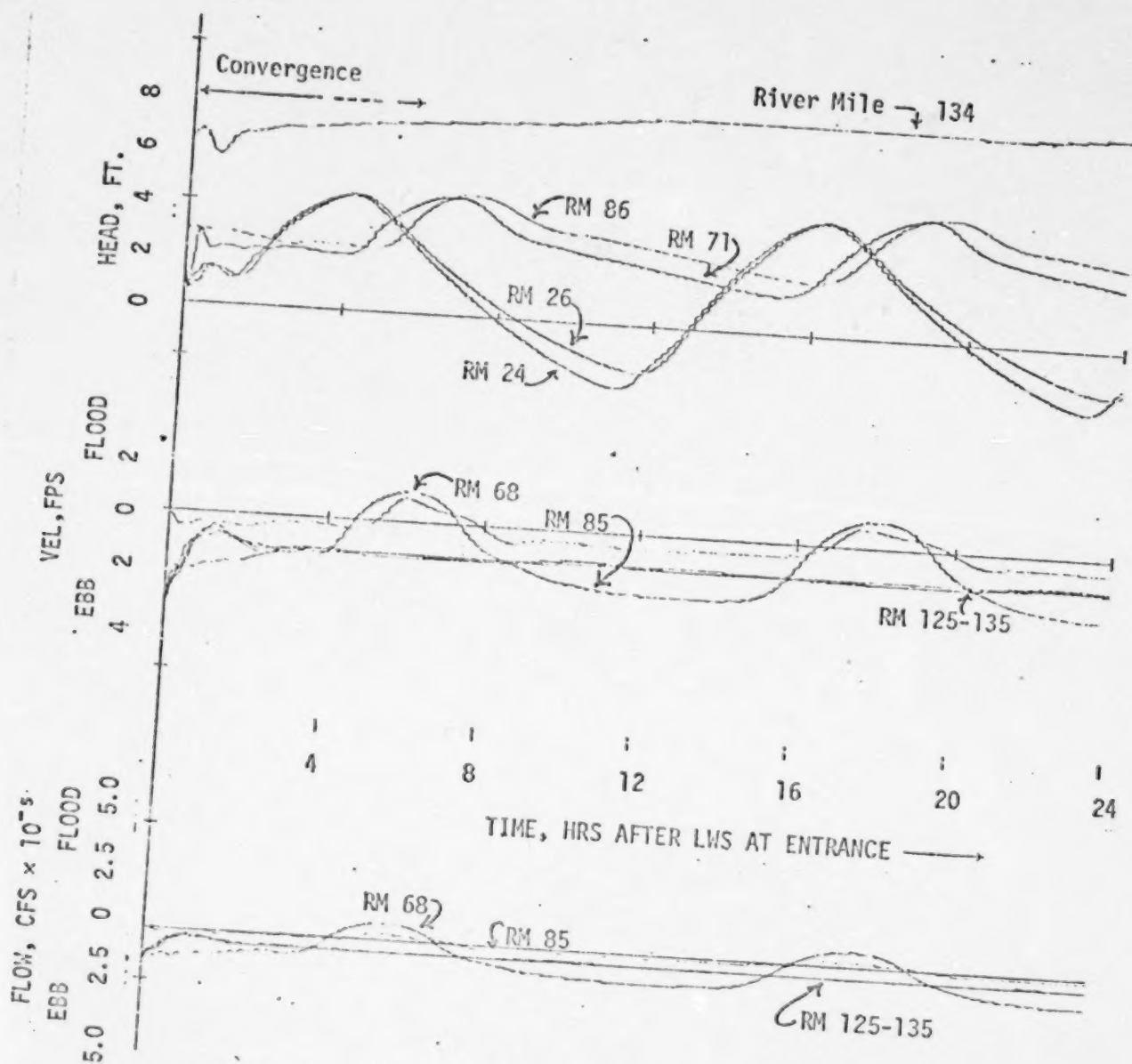


Fig. 13. Computed tidal elevation, velocity and flow. River miles 24-134, Columbia River, low flow conditions. From Callaway and Byram (1971).

FLUSHING

The residence time of pollutants in estuaries depends primarily on the freshwater runoff to the estuary. Calculation of the time required to remove dissolved substances results in a flushing rate and/or time. If the pollutant is discharged in or near the primary freshwater source, then the salinity distribution can be used for this computation; if not the distribution of the pollutant may have to be used.

Machine Calculations of Flushing

Once dynamic steady-state has been achieved in a time-varying model, i.e., after the pollutant has been distributed by allowing the program to run long enough, the flushing of a continuous release can be easily observed. The given distribution at time t is simply used as the initial distribution ($t = 0$) on a second run; the waste input is set to zero and the concentration decrease followed as long as need be.

For an instantaneous release, one sets the initial distribution in a given section or grid of the model to a certain level with no further inputs and allows the program to run (assuming the hydraulic portion, if any, of the model has achieved convergence).

Point source releases cannot be exactly simulated with a finite sized grid system, for the instantaneous or continuous case, of course.

Direct Observation

The response of an estuary to a sudden increase in streamflow can be quite dramatic as seen in Figure 14, a time-series of salinity in the Yaquina River Estuary (Callaway et al., 1970). Rainfall in inches per day is shown in the upper inset and the combined runoff from the Yaquina

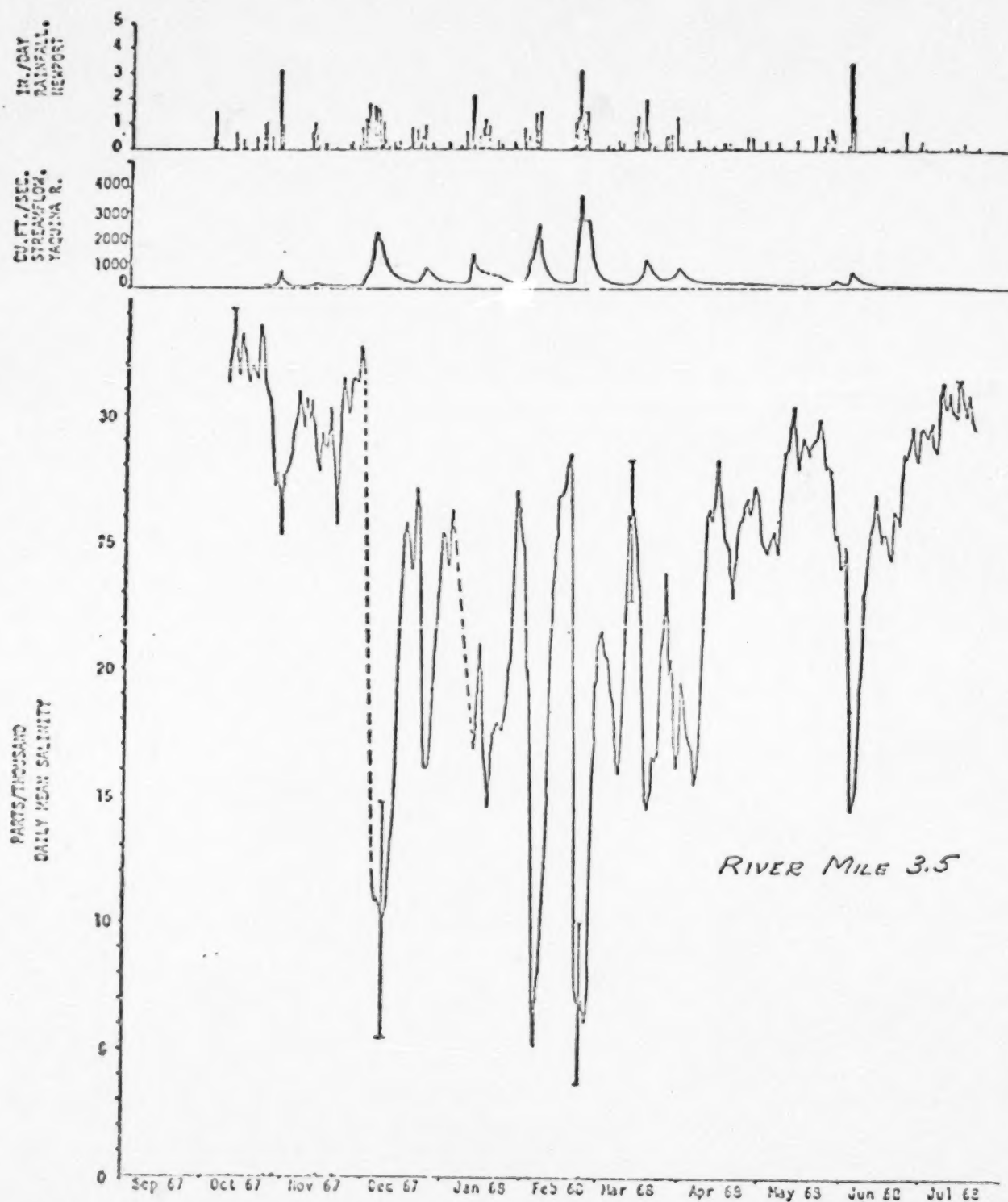


Fig. 14. Daily averages for selected parameters. From Callaway et al., (1969).

River and Elk Creek are shown below it. Coincident with the rainy season (starting in November, 1967) the salinity at mile 3.5 drops from about 33 ppt to 10 ppt.

With the succession of rainy periods during the winter, about 5 months are required until the higher salinity is achieved again (May, 1968). In June, 1968, a single intense storm of short duration lowered the salinity to about 15 ppt. It would appear from the hydrograph that local rainfall was effective in lowering the salinity as was the upstream runoff. In essence, then, flushing can be extremely fast and may at times sweep out the entire estuary.

Tidal Prism Method

The flushing rate of Grays Harbor was calculated by the tidal prism method described by Ketchum (1951). Here, the excursion length of the flood tide was used to divide the estuary into segments. Exchange ratios for each segment were calculated from:

$$r_n = \frac{P_n}{P_n + V_n}$$

where n = segment number

r_n = exchange ratio

P_n = tidal prism volume

V_n = low tide volume

Constant river flow was assumed (700 cfs) and a flushing time of 5.5 days was found.

The ratio of freshwater discharge during a half tidal cycle (12.4 hours) to P_n can be used to determine the type of vertical salinity distribution that is likely to occur:

$$\left\{ \int_0^T Q dt \right\} / P_n$$

where T = tidal period

Q = runoff

General, but not infallible, guides to vertical stratification were postulated by Schultz and Simmons (1957) who found that well-mixed estuaries have ratios of about 0.1; partly-mixed about 0.2-0.5; stratified conditions are found for values greater than 0.5 with 2-layer conditions being found at values near 1.0 or more. P_n/Q provides a measure of the flushing time.

Box Models

The "box" or "reservoir" model as used in oceans and estuaries has been described by Keeling and Bolin (1967) and Okubo and Pritchard (1969). In brief, one box or several connecting boxes are postulated as containing completely mixed fluids with exchange allowed within a box and between them at their mutual interface. Objections can be raised as to the loss of rigor when using the box model approach, i.e., replacing the appropriate differential equation with intuitive ideas on exchange processes and fluxes. However, when the system studied is complicated the box model method can prove a valuable tool, and perhaps the only tool, for studying processes between water masses and also between the water-air or water-sediment interface.

In a sense, all numerical models, i.e., finite sized grid models, are box models in that exchanges take place between grids at certain time steps. One difference between the "numerical" and the "box" model is that in the former case the governing equations are in terms of diffusion

coefficients while in the box model diffusion processes are expressed as rates. As Okubo and Pritchard (op. cit) put it, "The exchange rate constants in the box model have ... no clearly defined physical basis, but, to be honest about it, neither do the eddy diffusivities."

As an example, the box model approach is used for evaluation of the flushing rate of sulfite waste liquor (SWL) in Bellingham Bay. Flushing, as evidenced by a decrease in the surface concentration of SWL, can be accomplished by seaward advection of the surface layer, diffusion downward, and biodegradation. Data on the distribution of SWL emanating from a pulp mill in the northeast corner of Bellingham Bay have been described in U.S. Department of Interior (1967), which includes data collected by the author during and after a mill closure as shown in Figure 15. Collias et al. (1966) give an extensive analysis of the flushing mechanism in the Bay.

The Bay is considered to be a box with a depth, d , of 10 feet (the upper mixed layer), length, l , of 12 nautical miles, and width, b , of 5 nautical miles. Rates are expressed as (1) seaward advection through the south vertical plane, (2) vertical diffusion through the bottom horizontal plane, and (3) decay within the box.

The rate of change of SWL can thus be expressed as:

$$\frac{ds}{dt} = -\left(\frac{Q}{V} + \frac{K_z}{A} + K\right)s$$

where s = SWL concentration (ppm)

Q = net seaward advection ($L^3 T^{-1}$)

V = volume of the box (L^3)

K_z = vertical eddy diffusivity ($L^2 T^{-1}$)

A = area of the horizontal plane (L^2)

K = decay rate of SWL (T^{-1})

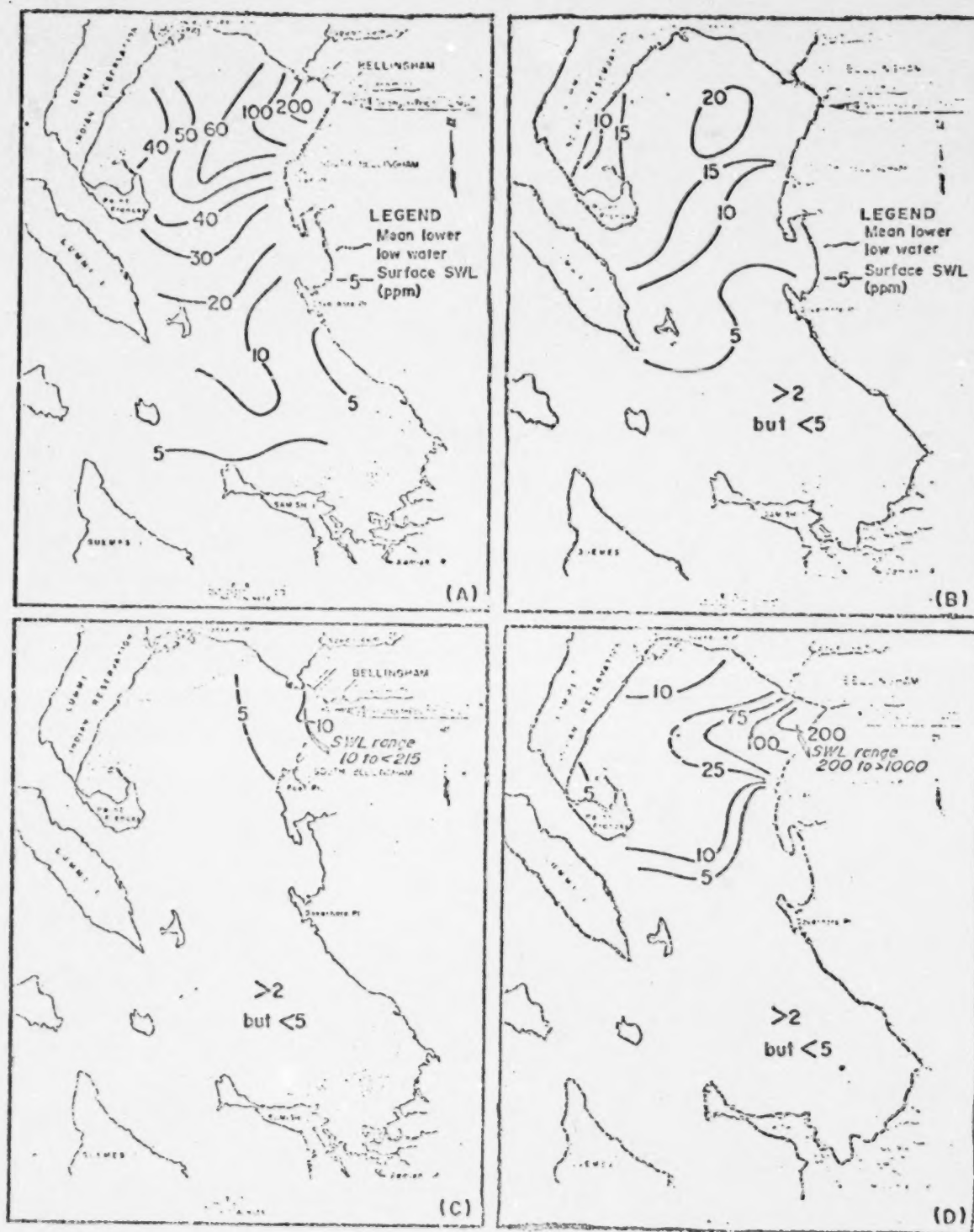


Fig. 15. Surface SWL: (A) average for the period Nov. 1959-Nov. 1961; (B) observed on Nov. 18, 1964; (C) observed on Nov. 25, 1964, and (D) observed on Dec. 1, 1964. From U. S. Dept. Int. (1967).

Seaward advection was estimated from net surface tidal velocities found at the entrances to the Bay by the USC&GS. Uncertainty exists as to the depth of the surface layer which for this example was chosen from an examination of vertical SWL profiles (Figure 16). Net velocities, passage widths, and depths result in a mean advective velocity, $\bar{U}$, of 0.1 ft. sec⁻¹. Since $Q = \bar{U} bd = 3.04 \times 10^4$ cfs, the rate of advection, $Q/V = 1.37 \times 10^{-6}$ sec⁻¹. The primary tributary, Nooksack River, has daily flows ranging between 595 and 46,200 cfs with a mean daily flow of 3,700 cfs. The latter value of Q would put $\bar{U}$ at 0.012 ft. sec⁻¹. Both estimates of Q will be used in the order of magnitude analysis.

Vertical flux was estimated from the steady-state expression:

$$\bar{U} \frac{\partial s}{\partial x} = K_z \frac{\partial^2 s}{\partial z^2}$$

and K_z was computed from:

$$K_z = (\bar{U} \frac{\partial s}{\partial x}) / (\frac{\partial^2 s}{\partial z^2})$$

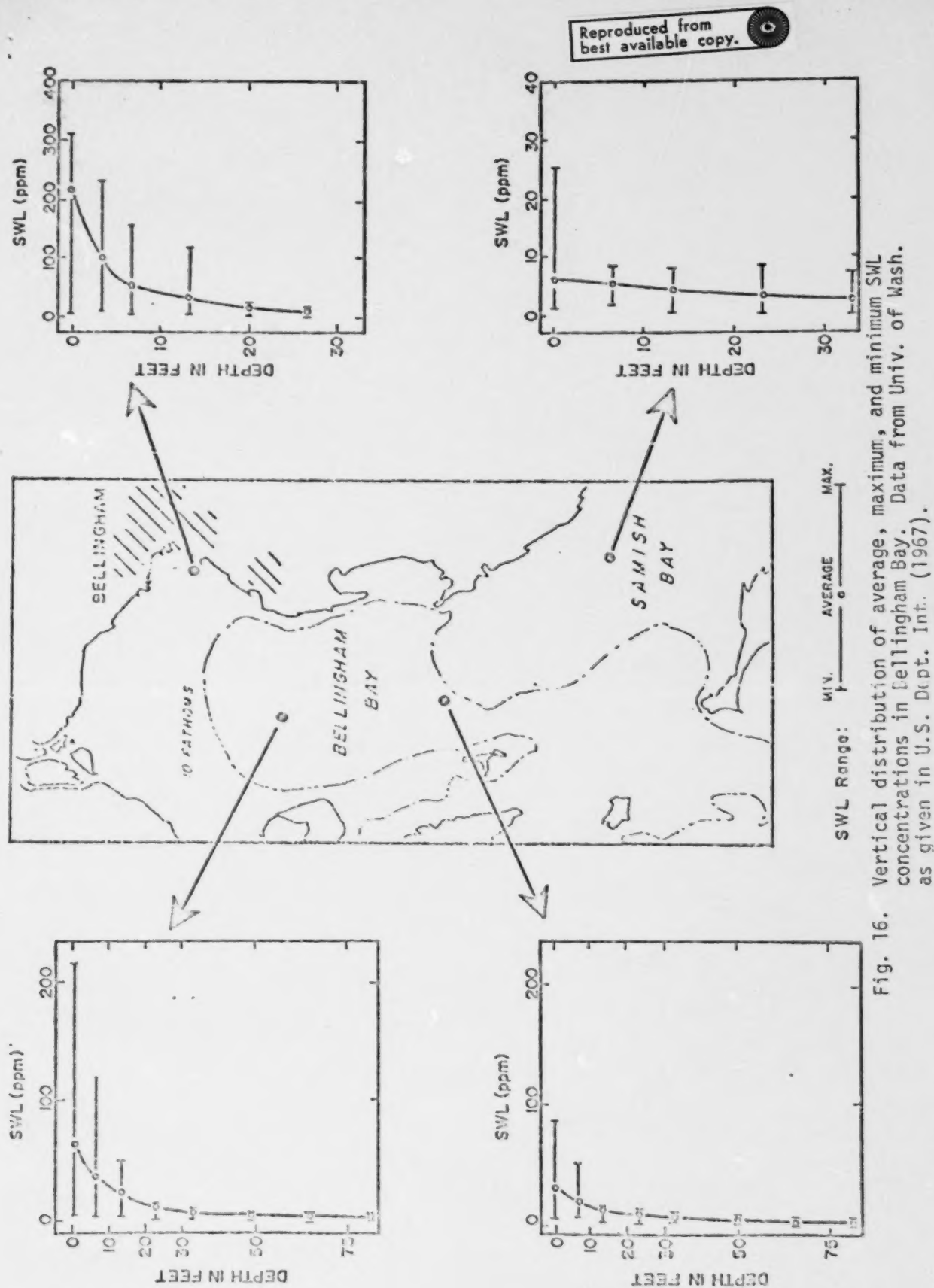
The horizontal SWL gradient, $\partial s / \partial x$, was found to be 0.009 ft.⁻¹, and the vertical term, $\partial^2 s / \partial z^2$, to be 0.06 ft.⁻², as displayed in Figures 15 and 16. For $\bar{U} = 0.1$, $K_z = 0.015$ ft² sec⁻¹. The half-life ($t_{1/2}$) of sulfite waste liquor in lagoons has been reported as 1 week by Lindsay et al. (1960). Assuming first order decay, the rate constant can be found from the expression $K = (t_{1/2})^{-1} \ln 0.5$.

For the lower advection rate ($Q = 3,700$ cfs) the magnitudes of the terms are:

$$Q/V = 0.17 \times 10^{-6} \text{ sec}^{-1}$$

$$K_z/A = 6.09 \times 10^{-11} \text{ sec}^{-1}$$

$$K = 1.09 \times 10^{-6} \text{ sec}^{-1}$$



From this analysis it can be seen that the loss by diffusion downward (for the box depth used) can be neglected relative to the other processes. The loss of measureable SWL by decay cannot be neglected at the flows used; however, as the size of the box decreases, K becomes negligible and Q/V becomes dominant.

Writing equation 18 in terms of the half-life of the distribution, taken as the time when the area (rather than the concentration) enclosed by the 5 ppm contour decreases to half that of the initial area:

$$\left(\frac{Q}{V} + K\right) = \frac{0.693}{t_{1/2}}$$

Upon substitution it is found that $t_{1/2} = 6.4$ days for $Q = 3700$ cfs. For $Q = 24000$ cfs, $t_{1/2} = 3.6$ days. For $Q = 48000$ cfs (the upper limit observed) $t_{1/2} = 6$ hours. The half-life found by planimetering the areas in Figure 15 was 2 days. The computed value seems reasonable considering the assumptions made.

In an attempt to calculate the rate, the word of warning mentioned at the beginning of this section was violated in obtaining a figure for the advection rate, namely, that the source of the effluent and the primary flushing source were in the same location. This is probably no worse a violation than the other assumptions such as complete mixing, etc.

In summary, it can be seen that a rapid estimate of flushing can be obtained via the box model approach but the data demands are no less stringent than a more sophisticated approach.

EFFECTS OF ESTUARIES ON OCEANS

The freshwater entering a river system eventually enters an estuary and is then discharged to sea. En route, losses may occur through seepage, evaporation or withdrawals, and agricultural or other diversions.

Water originally very pure in the headlands may, on its passage to the sea, accumulate substances which are detrimental. Some of the materials transported in this manner may precipitate out upon mixing with saltwater in the estuary, or be lost by deposition, by uptake of organisms, by evaporation, etc. Eventually, however, discharge of substances to the ocean will occur. When runoff is large, the plume of freshwater can be traced far offshore by observing the salinity distribution as shown for the Columbia, for example, by Duxbury et al. (1968). Work on the coastal distribution of Zn^{65} suggests that the best time to study its effects and to separate the Columbia from other coastal river discharges is during the winter when the Columbia is confined nearshore and directed northward; razor clam uptake of Zn^{65} is high at this time. At other times the Columbia plume is directed offshore.

Except in well-mixed estuaries, net flow landward along the bottom will occur; balancing the outflow in the surface layers by advection and entrainment of saline waters into the fresher surface layer. Offshore waters at depth will move inshore towards an estuary (the dynamics of this situation are different than these due to wind-related coastal upwelling) as has been demonstrated experimentally on the west coast for the Columbia River and San Francisco Bay; it appears to be a

reasonable assumption for Cook Inlet waters. Coincident with this net shoreward movement are pollution problems associated with offshore dumping and barge disposal (see Callaway, 1970).

Tamai et al. (1969) exhibited the variation of salinity concentration in the Pacific Ocean resulting from the jet action of the Columbia during the spring and summer as a plot of ratios of seawater, jet axis, and river mouth salinities versus the dimensionless ratio of axis distance (x) divided by the width of the river mouth (D_0). These data are plotted in Figure 17 as is the relation found by Jen et al. (1966) for comparison. The latter was derived from laboratory experiments to determine the characteristics of a warm water jet discharged horizontally into initially resting deep water; D_0 was 0.036 feet.

Pak et al. (1970) have presented data on light-scattering measurements in the Columbia River plume ($D_0 = 2$ n. miles) and were able to trace the plume for 200 km; they deduced that the scatterers were contained in the plume water for at least 30 days. Their data are also plotted on the figure.

The salinity distribution along the south coast of Alaska and Shelikof Island ($D_0 = 30$ n. miles) and the Aleutian Island chain was calculated in a like manner and is shown in the figure for the summer of 1957 (Callaway, 1963) and 1958 (Dodimead et al., 1963). Average combined river flow of the Susitna River and from Knik Arm in Cook Inlet are reported (Wagner et al., N.D.) as about 52,800; 102,000; 124,000; and 111,000 cfs during May, June, July, and August, respectively. The June through August values are in the range of low Columbia River flows.

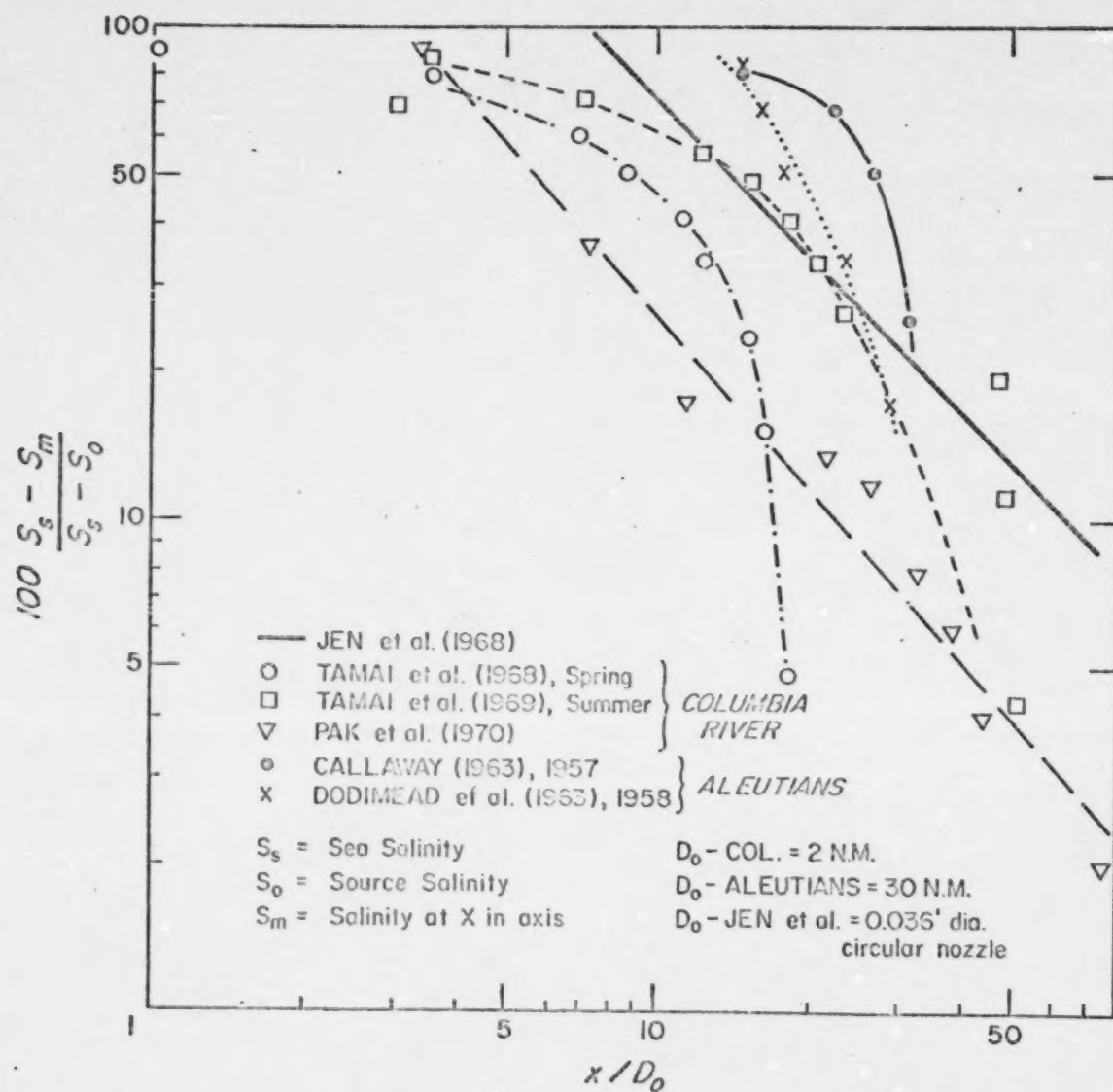


Fig. 17. Salinity or particle scattering along the jet axes of various water bodies.

The slopes of the four cases are somewhat similar. The ratio in the ordinate does not begin to fall off for the Aleutians until x is about equal to $10 D_0$, unlike the Columbia, suggesting less horizontal and/or vertical entrainment with distance from the orifice. Since the coast and islands prevent entrainment of less saline waters to the north and since current direction south of the chain is into Bristol Bay, entrainment will be less than that of a plume directed into the ocean (as for the Columbia) where it can occur on both sides of the plume.

Less dramatic effects will be exhibited by estuaries fed by smaller river systems; however, it should be clear from the above brief discussion that the estuary and ocean are interacting systems with the potential to affect each other in adverse as well as beneficial ways. The upwelling model given for Grays Harbor provides one example of the oceans affect on the estuary.

DISCUSSION

A rather broad view of models has been presented with emphasis on the physical processes in estuaries. Of course, the real problems concern ecology and the effect of pollutants and natural substances on animal and plant communities.

Some examples of model use have been given ranging from a slide rule approximation of flushing to a time varying digital computer solution of an estuary-river system. Applications to Pacific Northwest estuaries serve to point out the usefulness of steady-state ways of looking at things while anticipating full scale, at least 2-dimensional, simulation of our rather small but highly complex systems. The main waters of Puget Sound have been avoided in this discussion for lack of actual computational examples rather than to suggest that they are uninteresting or unimportant. The Sound is anything but that; hopefully it is not a setting awaiting its overdue succession of disasters. Visions of one large, uncoordinated, city subdivided into an array of political fiefdoms spreading from Olympia to Canada occur.

Pollution problems are difficult to solve. The more we learn about what were once thought to be rather straightforward relationships the more mysteries we uncover. If the problems relating to our estuaries are to be solved in a short time span then recourse will necessarily have to be made to models of some sort. As before, the complexity of a model has no real bounds; we must utilize the simplest, largest scale, model possible to attack existing problems rather than devoting too much attention to a virtually limitless collection of microcosmic structures. Information exchange among practicing engineers

actually using models or requiring them to make management decisions and theoretical and applied engineers and scientists is a necessity if we are to continuously narrow the gap between overly simplified models (what is practical) and those of the forefront variety (what is possible).

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